BLIND SEPARATION OF AUTO-CORRELATED IMAGES FROM NOISY MIXTURES USING MRF MODELS

Anna Tonazzini, Luigi Bedini, Ercan E. Kuruoglu and Emanuele Salerno

Istituto di Scienza e Tecnologie dell'Informazione
Area della Ricerca CNR di Pisa
Via G. Moruzzi, 1, I-56124 PISA, Italy
<surname> @iei.pi.cnr.it

ABSTRACT

This paper deals with the blind separation and reconstruction of source images from mixtures with unknown coefficients, in presence of noise. We address the blind source separation problem within the ICA approach, i.e. assuming the statistical independence of the sources, and reformulate it in a Bayesian estimation framework. In this way, the flexibility of the Bayesian formulation in accounting for prior knowledge can be exploited to describe correlation within the individual source images, through the use of suitable Gibbs priors. We propose a MAP estimation method and derive a general algorithm for recovering both the mixing matrix and the sources, based on alternating maximization within a simulated annealing scheme. We experimented with this scheme on both synthetic and real images, and found that a source model accounting for correlation is able to increase robustness against noise.

1. INTRODUCTION

Blind source separation (BSS), which became an active research topic in signal processing in the last decade, has only very recently received attention in image processing and computer vision. It consists of separating a set of unknown signals from a set of linear mixtures of them, when no knowledge is available about the mixing coefficients. The most well-known application example of BSS is the so-called "cocktail party" problem in audio processing. Other applications include the removal of underlying artifact components of brain activity from EEG records, the search for hidden factors in parallel financial data series, and feature extraction or noise removal from natural images. In order to solve BSS, which is a severely ill-posed inverse problem, many techniques have been proposed so far. Among them, the Independent Component Analysis (ICA) methods are based on the assumption of mutual independence of the sources. Most of these methods were developed in the case of noiseless data, and differ from one another in the way they enforce independence [1] [2][3]. The strict relationships among the various methods have been investigated as well [4], and some fast and efficient algorithms have been proposed, such as FastICA [5]. However, all these algorithms perform poorly when noise affects the data. Recently, some work has been done to partially overcome this limitation. In particular, the noisy FastICA algorithm has been proposed [6], and an Independent Factor Analysis (IFA) method has been developed [7][8][9]. Nevertheless, while providing satisfactory estimates of the mixing matrix, these methods still produce noisy source estimates, due to the typical ill-conditioning of the mixing matrix. We believe that a way to jointly obtain robust estimates for both the mixing matrix and the sources is to incorporate into the problem the available information about auto-correlation properties of the single sources. Indeed, correlation is an important feature of most real-world signals, and especially of images, and, when used as a constraint, it is known to be able to regularize and stabilize many ill-conditioned inverse problems.

In [10][11][12], Bayesian estimation has been proposed as a suitable, unifying framework for BSS, within which the other methods can be viewed as special cases. The Bayesian approach is also the most natural and flexible way to account for prior knowledge we may possess about a problem. Thus, in this paper we apply Bayesian estimation to regularize the blind separation of noisy mixtures of images. We retain the independence constraint of the ICA approach, and reformulate the BSS problem as the joint Maximun A Posteriori (MAP) estimation of the mixing matrix and the sources. The flexibility of the Bayesian formulation is exploited to incorporate constraints about the variables of the problem. In particular, Markov Random Fields (MRF) models, under the form of suitable stabilizing Gibbs priors, are used for both enforcing independence and describing the spatial correlation of the sources. We show how this modeling increases robustness of the estimates against noise. For the joint maximization of the posterior probability with re-

This work has been partially supported by the Italian Space Agency under contract ASI/CNR 1/R/073/01.

spect to all the unknowns, we propose a method based on an iterative alternating maximization scheme, where steps of MAP estimation for the sources alternate with steps of MAP estimation for the mixing matrix. In particular, we develop a general computational scheme, governed by a simulated annealing algorithm, which can cope with any kind of constraint on the mixing matrices and possibly non-convex, edge-preserving, MRF models for the sources. Depending on the specific constraints adopted, existing deterministic algorithms for convex or non-convex optimization can be used in place of the Metropolis algorithm, in order to reduce computational complexity.

2. BSS THROUGH ICA AND BAYESIAN ESTIMATION

According to the BSS formalism, the data generation model we consider is given by:

$$\mathbf{x}(t) = A\mathbf{s}(t) + \mathbf{n}(t) \qquad t = 1, 2, ..., T \tag{1}$$

where $\mathbf{x}(t)$ is the vector of the measurements, $\mathbf{s}(t)$ is the column vector of the unknown sources, and $\mathbf{n}(t)$ is the noise or measurement error vector, at location t, and A is the unknown mixing matrix. We assume the same number N of measured and source signals, so that A is an $N \times N$ matrix. For later use, we define also the vectors

$$\mathbf{s}_i = (s_i(1), s_i(2), ..., s_i(T)) \qquad i = 1, 2, ..., N$$

Considering a white and Gaussian noise with zero mean, the likelihood is given by:

$$P(\mathbf{x}|\mathbf{s}, A) = \frac{1}{Z_\Sigma} \times \tag{2}$$

$$\times \exp\left\{ -\frac{1}{2} \sum_t \left(A\mathbf{s}(t) - \mathbf{x}(t)\right)' \Sigma^{-1}(t) \left(A\mathbf{s}(t) - \mathbf{x}(t)\right) \right\}$$

where $\mathbf{x} = (\mathbf{x}(1), ..., \mathbf{x}(T))$, $\mathbf{s} = (\mathbf{s}(1), ..., \mathbf{s}(T))$, Σ is the covariance matrix of the noise, assumed, in general, to be location-dependent, and Z_Σ is the normalizing constant. Note that the maximum likelihood principle applied to eq. 2 would clearly give an undetermined problem, unless more information is exploited. In a fully Bayesian approach, both A and $\mathbf{s}$ are assumed as independent unknowns, and are assigned with prior distributions $P(A)$ and $P(\mathbf{s})$, respectively. Then, A and $\mathbf{s}$ can be simultaneously estimated by maximizing some joint distribution, e.g. $P(\mathbf{s}, A, \mathbf{x})$. Our problem thus becomes:

$$(\hat{\mathbf{s}}, \hat{A}) = arg \max_{\mathbf{s}, A} P(\mathbf{s}, A, \mathbf{x}) \tag{3}$$

Note that $P(\mathbf{s}, A, \mathbf{x})$ can be decomposed as:

$$P(\mathbf{s}, A, \mathbf{x}) = P(\mathbf{x}|\mathbf{s}, A)P(\mathbf{s})P(A) = P(\mathbf{s}, A|\mathbf{x})P(\mathbf{x}) \tag{4}$$

Apart from $P(\mathbf{x})$, which depends neither on A nor on $\mathbf{s}$, $P(\mathbf{s}, A, \mathbf{x})$ is thus equal to the posterior distribution $P(\mathbf{s}, A|\mathbf{x})$. Hence, problem of eq. 3 is equivalent to the following MAP estimation problem:

$$\begin{aligned} (\hat{\mathbf{s}}, \hat{A}) &= arg \max_{\mathbf{s}, A} P(\mathbf{s}, A, |\mathbf{x}) = \\ &= arg \max_{\mathbf{s}, A} P(\mathbf{x}|\mathbf{s}, A)P(\mathbf{s})P(A) \end{aligned} \tag{5}$$

The posterior distribution, or equivalently distribution $P(\mathbf{s}, A, \mathbf{x})$, accounts for all information we have about the problem, and can restrict the set of solutions associated to the likelihood part of eq. 5 by means of the priors $P(\mathbf{s})$ and $P(A)$. In the ICA approach A is assumed to have a uniform prior and the sources are assumed only to be mutually independent, that is:

$$P(\mathbf{s}) = \prod_i P_i(\mathbf{s}_i) \tag{6}$$

whatever the form of the marginals P_i. If all the P_i's are equal, the well-known permutation and scale indeterminacies of the ICA solution arise. The presence of the prior $P(A)$ in eq. 5 may contribute in restricting the set of solutions to our problem. In particular, any prior which is sensitive to column interchanges eliminates the permutation indeterminacy. If $P(A)$ is able to constrain the values of the elements of A, the scale indeterminacy is also eliminated. In general, the construction of an objective function like the one in eq. 5 allows us to include any possible prior knowledge on $\mathbf{s}$ and A, and this is not so immediate in the ICA approach. In this paper, we assume independent sources, thus letting eq. 6 to hold true, but adopt Markov Random Field models to describe regularity properties, under the form of spatial correlation (smoothness), for the individual sources. This will be shown to make the Bayesian approach advantageous over ICA in terms of robustness against noise.

Nevertheless, as usual in blind signal/image estimation, joint maximization eq. 5 is very hard, and needs to be reduced in some way. In [13] we proposed to solve eq. 5 via an Expectation Maximization (EM) algorithm. However, it is well known that EM is usually plagued by local extrema, and requires problem-dependent approximations, such as mean field and saddle point approximation, to manage expectations. Another popular strategy to perform joint MAP estimations consists in iteratively alternating steps of estimation with respect to the different sets of variables. In our case, this results in steps of estimation of A and $\mathbf{s}$, respectively:

$$A^{(k)} = arg \max_{A} P(\mathbf{x}|\mathbf{s}^{(k-1)}, A)P(A) \tag{7}$$

$$\mathbf{s}^{(k)} = arg \max_{\mathbf{s}} P(\mathbf{x}|\mathbf{s}, A^{(k)})P(\mathbf{s}) \tag{8}$$

For general forms of the prior adopted for A, problem of eq. 7 can be non-concave, and algorithms for non-convex optimization must be adopted, such as simulated annealing (SA). In this case, however, owing to the small number of variables, even SA results to be a reasonably cheap algorithm. Moreover, SA can be particularly suitable when $P(A)$ enforces constraints on A that cannot be expressed in analytical form, such as bounds on the admissible values, etc. The algorithm to be adopted to solve the optimization problem of eq. 8 mainly depends on the form adopted for the $P_i(\mathbf{s}_i)$. On the other hand, we know from the ICA theory that source separation (at least in the noiseless case) is only possible when at most one of the sources is Gaussian. Thus, in general, problem of eq. 8 will not be quadratic. However, when appropriate, the various $P_i(\mathbf{s}_i)$ can be chosen so to ensure the concavity of the function to be optimized, so that a gradient ascent algorithm can be used. Otherwise, still relatively cheap algorithms for non-convex optimization can be used, such as Graduated Non-Convexity (GNC) [14], which was shown to be computationally more efficient than SA. In Section 3 we will show that the constraint we intend to enforce on the sources, i.e. spatial correlation, can be expressed through MRF models whose related Gibbs distributions are suitable to be managed by GNC-like algorithms. On the basis of this choice, we will specialize the iterative scheme of eqs.7-8 and propose a particular implementation of the scheme itself that exhibits a reduced computational complexity and for which we experimentally verified that convergence can always be obtained.

3. CHOICE OF THE GIBBS SOURCE PRIORS

MRF models have become very popular since the middle '80s, especially in connection to inverse problems of image processing, such as restoration, denoising, segmentation, optical flow estimation, and so on. Through MRF models, it is indeed possible to describe local properties of the images, such as edges, in order to make the regularizing smoothness constraint to become space-variant. Furthermore, the local nature of these models allows us to devise distributed and parallel algorithms. In this paper we propose to use MRF in a BSS and ICA context, for modeling both the independence among the sources and the local properties of spatial correlation of each source. Let us consider then the distribution of the $i-th$ source $s_i(t)$ in our problem. According to the MRF formalism, it must have the following Gibbs form:

$$P_i(\mathbf{s}_i) = \frac{1}{Z_i}\exp\{-U_i(\mathbf{s}_i)\} \tag{9}$$

where Z_i is the normalizing constant and $U_i(\mathbf{s}_i)$ is the prior energy in the form of a sum of potential functions over the set of cliques of interacting locations. We consider the set of cliques constituted of a single location t or two adjacent locations t and r, in the 2D grid of the image. We then define $U_i(\mathbf{s}_i)$ as:

$$U_i(\mathbf{s}_i) = \alpha_i \sum_{t=1}^{T} f_i(s_i(t)) + \beta_i \sum_{\{t,r\}} \phi_i\left(s_i(t) - s_i(r)\right) \tag{10}$$

where α_i and β_i are two positive weights, and f_i and ϕ_i are two functions to be chosen according to our expectation about the probability law assigned to each sample of signal $\mathbf{s}_i$, and about the degree of correlation we assign to couples of adjacent samples, respectively. Unless more precise knowledge about their nature is available, in many practical ICA algorithms the log probability distribution f_i of the source samples is approximated via arbitrary sub-Gaussian or super-Gaussian distributions. Function ϕ_i should instead describe some regularity in the $i-th$ source, by penalizing high gradient values. This regularity is physically plausible in many real-world applications, and, as already said, it is an essential constraint to prevent the reconstructions from being unstable when the data are noisy. Nevertheless, the source signals can present some steep fronts which must be preserved as well. We thus refer to stabilizers for edge-preserving image recovering [15]. Among the many proposed, some stabilizers possess the characteristic of being convex [16], therefore, if the f_i are also convex, the function to be maximized in step 8 will result concave, and a gradient ascent can be used to perform the optimization. In the most general case of non-concavity, specific stabilizers were proposed that allows us to "correct" the mild non-concavity of the overall function by providing a sequence of approximations for it, to be optimized in turn according to the GNC strategy. In particular, we experimented an edge-preserving model for which we previously derived a family of approximations and a GNC-like algorithm [17]. The general form of the related function ϕ is:

$$\phi(\xi) = \frac{\lambda|\xi|/\Delta}{1 + |\xi|/\Delta} \tag{11}$$

where parameter Δ is a threshold for the intensity gradient above which a steep front is likely to be present in the signal, and parameter λ is chosen accordingly to the desired degree of smoothness, which is related, in its turn, to the image scale and the amount of noise. The choice of these parameters has thus to be done on the basis of prior knowledge. Methods for their automatic data-driven selection can also be found. In [18] [19], we developed a Bayesian technique for the joint estimation of the image and the MRF model parameters, based on the data alone, in a context of blind deconvolution. This technique could be extended in a straightforward manner to the present BSS problem.

We also experimented a convex edge-preserving stabilizer [20], in order to perform source estimation via conjugate gradient. The general form of function ϕ is:

$$\phi(\xi) = \begin{cases} \lambda \xi^2 & \text{if } |\xi| \leq \Delta \\ \lambda \left(2\Delta|\xi| - \Delta^2\right) & \text{if } |\xi| > \Delta \end{cases} \tag{12}$$

where the parameters have the same meaning as before. We found however that, in all cases, this stabilizer has weaker edge-preserving properties, and gives convergence problems.

4. DERIVATION OF THE ALTERNATING MAXIMIZATION ALGORITHM

Upon the considerations made above about the choice of the source models, we propose a particular implementation of the iterative scheme of eqs. 7-8 which is able to further reduce the computational complexity and ensure convergence. This scheme is based on an overall simulated annealing for the estimation of A according to 7, interrupted at each cycle, i.e. at each lowering of a temperature parameter τ, to perform an update of the sources $\mathbf{s}$, according to eq. 8. To this end, we first take the negative logarithm of the distribution $P(\mathbf{x}|A,\mathbf{s})P(\mathbf{s})P(A)$, thus obtaining the following energy function to be minimized in A and $\mathbf{s}$:

$$E(\mathbf{s},A) = \frac{1}{2}\sum_{t=1}^{T} \left(A\mathbf{s}(t) - \mathbf{x}(t)\right)' \Sigma^{-1}(t) \left(A\mathbf{s}(t) - \mathbf{x}(t)\right) + \\ + \sum_i \sum_{t=1}^{T} \alpha_i f_i(s_i(t)) + \sum_i \sum_{\{t,r\}} \beta_i \phi_i \left(s_i(t) - s_i(r)\right) + \\ -logP(A) \tag{13}$$

where the constant terms coming from the partition functions have been neglected. A new distribution $P_\tau(\mathbf{s},A)$, to be used for simulated annealing, can thus be derived in the following way:

$$P_\tau(\mathbf{s},A) = \frac{1}{Z} exp\left\{-\frac{E(\mathbf{s},A)}{\tau}\right\} \tag{14}$$

The general scheme of our algorithm is the following:

1. set $k = 0$, $\mathbf{s}^{(k)}$, $A^{(k)}$, τ_k
2. set $r = 1$, $A^{(r)} = A^{(k)}$; for $r = 1, ..., L$

   compute $A^{(r+1)}$ according to $P_{\tau_k}(\mathbf{s}^{(k)}, A)$

3. set $A^{(k+1)} = A^{(L)}$
4. compute

   $$\mathbf{s}^{(k+1)} = arg \min_{\mathbf{s}} E(\mathbf{s}, A^{(k+1)})$$

5. set $k = k + 1$; go back to step 2 until a termination criterion is satisfied.

In step 2, the stochastic updating of the mixing matrix is performed via a Metropolis algorithm, giving random, normally distributed, perturbations to each matrix coefficient by turns. The whole matrix is visited L times for each temperature value. In the scheme above, the lowering to zero of the temperature ensures the convergence of the estimation of the mixing matrix A, in that distribution in eq. 14 becomes a Dirac function when τ approaches zero. This ensures stabilization of the source estimates as well, since these are computed by minimizing the energy function with all fixed parameters.

As previously highlighted, the above procedure is suitable to be augmented with a step of estimation of the MRF model parameters.

5. EXPERIMENTAL RESULTS

The efficiency of the alternating maximization algorithm described in Sections 4 was tested on piecewise smooth synthetic and real images. For generating The synthetic images were generated without referring to any specific probability law, and, in this first application of the method, we enforced only generic constraints of spatial correlation for the sources, thus dropping the first term from the prior energy of eq. 10. Similarly, we did not assume any a priori knowledge about the mixing matrix coefficients. As for the stabilizer, in the absence of specific knowledge about the single sources, we assumed the same stabilizer of eq. 11 for each of them, with same parameters. It is however to be noted that, when appropriate, assigning different stabilizers/parameters to the different sources permits to avoid the typical permutation ambiguity of BSS, and we verified that the order in which we assign the prior to the sources is reflected in the order of the reconstructions [21]. In all the experiments, the mixtures were generated numerically, by letting the ideal matrix coefficients and the noise realization to be selected randomly. The starting point for the mixing matrix was always chosen randomly, while the starting point for the sources was always set to the mixtures.

In a first set of experiments, we considered synthetic images. Figure 1 shows the typical results we obtained when considering two simple piecewise constant images, whose mixtures were added with a space-invariant noise (SNR=26 dB). Trying with several randomly selected mixing matrices and several noise realizations, we always obtained similar results in terms of RMSE between the original and estimated matrix and sources. Since we adopted the same stabilizer with same parameters λ and Δ for the two images, for some choices of the mixing matrix we obtained the reconstructed images in the reversed order. Figures 1a-1b show the mixtures, and Figure 1b-1c show the demixed images,

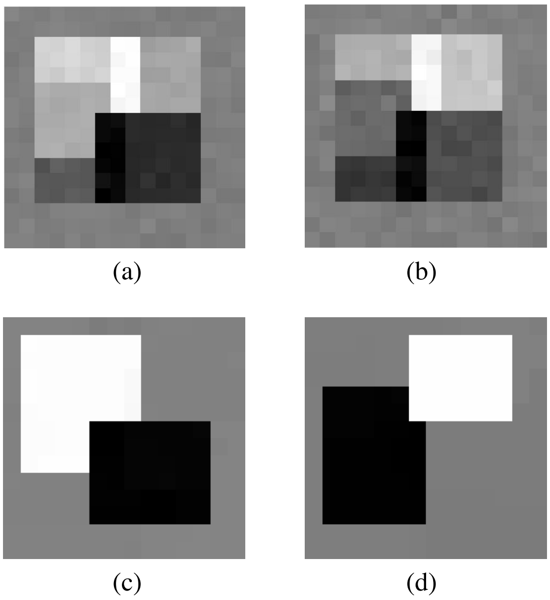


Fig. 1. Synthetic images: (a) first mixture (SNR=26 dB); (b) second mixture (SNR=26 dB); (c) first demixed image; (d) second demixed image.

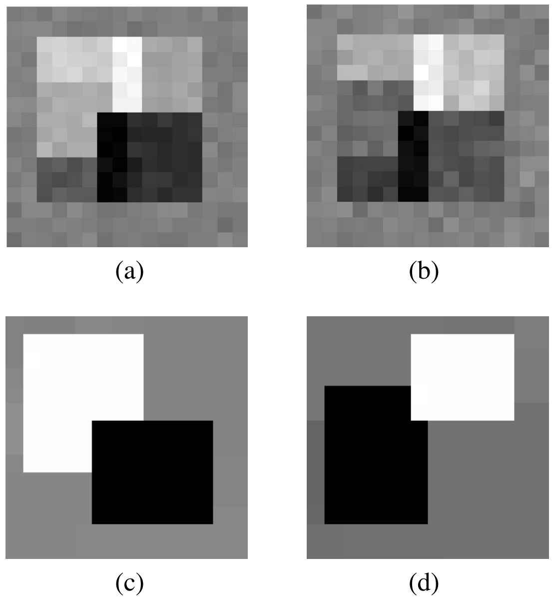


Fig. 2. Synthetic images: (a) first mixture (SNR=20 dB); (b) second mixture (SNR=20 dB); (c) first demixed image; (d) second demixed image.

for the following original mixing matrix:

$$A = \begin{bmatrix} 0.6998 & 0.3005 \\ 0.2977 & 0.4026 \end{bmatrix}$$

The recovered matrix was in this case:

$$\hat{A} = \begin{bmatrix} 0.6961 & 0.3020 \\ 0.2901 & 0.4002 \end{bmatrix}$$

obtained with $\lambda = 0.1$ and $\Delta = 2$. It is to be noted that for these synthetic images the FastICA algorithm is not able to separate the mixture in all the cases we considered. This fact is probably due to the small number of levels present in the original images, and could mean that our method is particularly suitable to robustly separate noisy mixtures of, say, binary signal/images. A possible application could be, for instance, bleed-through cancellation in document images. We also attempted with higher noise levels, and we obtained still satisfactory separations, although we observed an obvious degradation as the noise level increases. For instance, for the same original sources and matrix as before, when the signal-to-noise ratio is 20 dB, with $\lambda = 1$ and $\Delta = 2$ we obtained the reconstructions shown in Figure 2, and the following mixing matrix:

$$\hat{A} = \begin{bmatrix} 0.6953 & 0.2953 \\ 0.2813 & 0.4062 \end{bmatrix}$$

Note that here, since the noise is higher than in the previous case, we adopted a higher value for parameter λ.

The performance of the method on real images, artificially mixed, was similar to the one described for the synthetic images. Figure 3 shows one of the results obtained for a signal-to-noise ratio of 26 dB, when the original mixing matrix was:

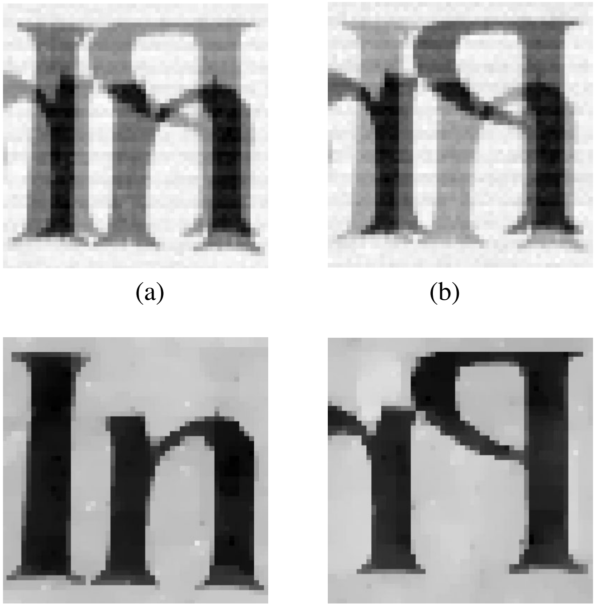
$$A = \begin{bmatrix} 0.8214 & 0.6154 \\ 0.4447 & 0.7919 \end{bmatrix}$$

The recovered matrix, for $\lambda = 1$ and $\Delta = 2$, was in this case:

$$\hat{A} = \begin{bmatrix} 0.8215 & 0.5720 \\ 0.4333 & 0.7412 \end{bmatrix}$$

6. CONCLUSIONS

We proposed a Bayesian formulation of ICA techniques for the blind source separation of noisy mixtures of images. We considered MRF models for the sources which are suitable to describe both the independence of the sources themselves and the local spatial correlation for each single source. Among the various available MRF models, we considered edge-preserving models, either convex or non-convex, for which deterministic optimization algorithms have been established. We proposed an alternating maximization scheme for the joint MAP estimation of the mixing matrix and the sources. In the general case where a priori information is available on the mixing matrix, alternating maximization can be implemented through an overall simulated annealing scheme,



(a) (b) (c) (d)

Fig. 3. Real images: (a) first mixture (SNR=26 dB); (b) second mixture (SNR=26 dB); (c) first demixed image; (d) second demixed image.

where the Metropolis algorithm is employed for updating the mixing matrix, and a deterministic algorithm is employed for updating the sources, at each temperature. We experimentally verified on both synthetic and real and images that, even in absence of priors for A, the introduction of information about the spatial correlation of the sources can increase robustness of the estimates against noise.

7. REFERENCES

[1] A.J. Bell and T.J. Sejnowski, ”An information maximization approach to blind separation and blind deconvolution”, Neural Computation, vol.7, pp. 1129-1159, 1995.

[2] T. Lee, M. Lewicki and T. Sejnowski, ”Independent component analysis using an extended infomax algorithm for mixed sub-gaussian and super-gaussian sources”, Neural Computation, vol. 11, pp. 409-433, 1999.

[3] J.-F. Cardoso, ”High-order contrasts for independent component analysis”, Neural Computation, vol.11, pp. 157-192, 1999.

[4] S. Amari and A. Cichocki, ”Adaptive blind signal processing - neural network approaches”, Proc. IEEE, vol. 86, pp. 2026-2048, 1998.

[5] A. Hyvarinen, ”Fast and Robust Fixed-Point Algorithms for Independent Component Analysis”, IEEE Trans. NN, vol.10, pp. 626-634, 1999.

[6] A. Hyvarinen, ”Gaussian moments for noisy independent component analysis”, IEEE Signal Processing Letters, vol. 6, pp. 145-147, 1999.

[7] H. Attias, ”Independent Factor Analysis”, Neural Computation, vol. 11, pp. 803-851, 1999.

[8] E. Moulines, J. F. Cardoso, E. Gassiat, ”Maximum likelihood for blind separation and deconvolution of noisy signals using mixture models”, Proc. ICASSP’97, vol. 5, pp. 3617-3620, 1997.

[9] E. Kuruoglu, L. Bedini, M.T. Paratore, E. Salerno, A. Tonazzini, ”Source separation in astrophysical maps using independent factor analysis”, Neural Networks, to appear, 2003.

[10] K. Knuth, ”Bayesian source separation and localization”, Proc. SPIE’98, Bayesian Inference for Inverse Problems, pp. 147-158, 1998.

[11] S. E. Lee and S.J. Press, ”Robustness of Bayesian factor analysis estimates”, Communications in Statistics - Theory and Methods, vol.27, 1998.

[12] A. Mohammad-Djafari, ”A Bayesian approach to source separation”, Proc. 19th Int. Workshop on Maximum Entropy and Bayesian Methods, 1999.

[13] A. Tonazzini, L. Bedini, E. E. Kuruoglu and E. Salerno, ”Blind separation of time-correlated sources from noisy data”,Tech. Rep. IEI-CNR No. TR-42-2001, 2001.

[14] L. Bedini, I. Gerace, E. Salerno and A. Tonazzini, ”Models and algorithms for edge-preserving image reconstruction”, in: Advances in Imaging and Electron Physics, P.W. Hawkes Ed., Academic Press, vol. 97, pp. 86-189, 1996.

[15] S. Z. Li, ”On discontinuity-adaptive smoothness priors in computer vision”, IEEE Trans. PAMI, vol. 17, pp. 576-586, 1995.

[16] S.Z. Li, ”Close-form solution and parameter selection for convex minimization-based edge-preserving smoothing”, IEEE Trans. PAMI, vol. 20, pp. 916-932, 1998.

[17] L.Bedini, I.Gerace and A.Tonazzini, ”A GNC algorithm for constrained image reconstruction with continuous-valued line processes”, Pattern Recognition Letters, vol. 15, pp. 907-918, 1994.

[18] L. Bedini, A. Tonazzini and S. Minutoli, ”Unsupervised edge-preserving image restoration via a saddle point approximation”, Image and Vision Computing, vol. 17, pp. 779-793, 1999.

[19] L. Bedini and A. Tonazzini, ”Fast fully data driven image restoration by means of edge-preserving regularization”, Real-Time Imaging, vol.7, pp. 3-19, 2001.

[20] D. Shulman and J.Y. Herve, ”Regularization of discontinuous flow fields”, Proc. Workshop on Visual Motion, pp. 81-86, 1989.

[21] L. Bedini, E. Salerno, A. Tonazzini, ”Blind Source Separation from noisy data using Bayesian Estimation and Gibbs Priors”, Proc. IASTED-SIP’2001, pp. 108-112, 2001.