BLIND SEPARATION OF MULTIPLE CONVOLVED COLORED SIGNALS USING SECOND-ORDER STATISTICS

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ABSTRACT

The present paper deals with the blind separation of multiple convolved colored signals, that is, the blind deconvolution of an Multiple-Input Multiple-Output Finite Impulse Response (MIMO-FIR) system. To deal with the blind deconvolution problem using the second-order statistics (SOS) of the outputs, Hua and Tugnait [3] considered it under the conditions that **a)** the FIR system is irreducible and **b)** the input signals are spatially uncorrelated and have distinct power spectra. In the present paper, the problem is considered under a weaker condition than the condition **a)**. Namely, we assume that **c)** the FIR system is equalizable by means of the SOS of the outputs [6]. Under **b)** and **c)**, we show that the system can be blindly identified up to a permutation, a scaling, and a delay using the SOS of the outputs. Moreover, based on this identifiability, we show a novel necessary and sufficiently condition for solving the blind deconvolution problem, and then, based on the condition, we propose a new algorithm for finding an equalizer using the SOS of the outputs.

1. INTRODUCTION

The present paper deals with the blind separation of multiple convolved colored signals, that is, the blind deconvolution of MIMO-FIR systems driven by colored signals. Hua and Tugnait [3] showed that an MIMO-FIR system driven by colored signals is identifiable up to a permutation and a scaling using the SOS of the outputs under the conditions a) and b) (shown in the abstract).

One of the objectives of the present paper is to show that even if the MIMO-FIR system is assumed to satisfy the condition **c)**, then under the condition **b)**, the system can be identifiable up to a permutation, a scaling and a delay using the SOS of the outputs. Besides, based on this identifiability, we show a necessary and sufficiently condition for solving the blind deconvolution (see Theorem 2 in Section 3). This condition is a novel key point in the present paper. Moreover, based on the condition we propose a new algorithm to solve the blind deconvolution problem using the SOS of the output signals under the conditions **b)** and **c)**. Hua, et al. [4] proposed an indirect method for solving the blind deconvolution problem under the conditions **a)** and **b)**. Differently from the method, we consider a direct method. Namely, the proposed algorithm is derived from the steepest descent minimization of a non-negative function which takes a minimum only when the outputs are uncorrelated with each other, and works such that the colored inputs are recovered from the outputs. The non-negative function is derived from the *Hadamard Inequality* [2] and the proposed algorithm is implemented in the frequency domain. Wu and Principe [12] proposed an algorithm for the blind deconvolution of a convolutive mixture in the frequency domain, using the *Hadamard Inequality*. However, differently from the conventional algorithms in the frequency domain, our proposed algorithm has an attractive property where the cumbersome permutation problem usually arising in frequency domain approaches (e.g., [10, 12]) does not occur.

This paper uses the following notation: The superscripts T, $*$, and H denote the transpose, the complex conjugate, and the complex conjugate transpose (Hermitian) operations of a matrix, respectively. Let $\boldsymbol{D}$, $\boldsymbol{P}$, and $\boldsymbol{\Lambda}(z)$ denote, respectively, an $n \times n$ diagonal matrix with diagonal entries d_i $(i = 1, \cdots, n)$, an $n \times n$ permutation matrix, and an $n \times n$ regular diagonal matrix with diagonal entries being monic monomials z^{l_i} $(i = 1, \cdots, n)$, where l_i is a non-negative integer. Let $det\boldsymbol{X}$ denote the determinant of a matrix $\boldsymbol{X}$. Let $\boldsymbol{X}_H(z)$ denote the para-Hermitian conjugate of $\boldsymbol{X}(z)$, that is, $\boldsymbol{X}_H(z) = \boldsymbol{X}^*(1/z^*)$ [7]. For example, if $X(z) = 1 + az$, then $X_H(z) = 1 + a^* z^{-1}$.

2. PROBLEM FORMULATION

The following MIMO-FIR system is considered as a transmission system between sources and sensors in the present paper:

$$\boldsymbol{x}(t) = \sum_{k=0}^{K-1} \boldsymbol{H}^{(k)} \boldsymbol{s}(t-k) + \boldsymbol{n}(t), \tag{1}$$

where $\boldsymbol{x}(t)$ represents an m-column output vector called the *sensor signal*, $\boldsymbol{s}(t)$ represents an n-column input vector called the *source signal*, $\boldsymbol{n}(t)$ is an m-column noise vector, $\{\boldsymbol{H}^{(k)}\}$ is an $m \times n$ matrix sequence representing the impulse response of the transmission system (channel), and the number K denotes its length. Equation (1) can be written as

$$\boldsymbol{x}(t) = \boldsymbol{H}(z)\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{2}$$

where $\boldsymbol{H}(z)$ is the z-transform of the impulse response, i.e.,

$$\boldsymbol{H}(z) = \sum_{k=0}^{K-1} \boldsymbol{H}^{(k)} z^k.$$

Note that the notation z is used instead of the commonly used z^{-1} in the z-transform.

Here, let us consider the following FIR system called an *equalizer* which is driven by the sensor signals.

$$\boldsymbol{y}(t) = \sum_{k=0}^{M-1} \boldsymbol{W}^{(k)} \boldsymbol{x}(t-k), \tag{3}$$

where $\boldsymbol{y}(t)$ is an n-column vector representing the output signal of the equalizer (i.e., the number of the outputs is equal to the number of the inputs of the transmission system.), $\{\boldsymbol{W}^{(k)}\}$ is an $n \times m$ matrix sequence, and the number M is the length of the equalizer. Equation (3) can be written as

$$\boldsymbol{y}(t) = \boldsymbol{W}(z)\boldsymbol{x}(t), \tag{4}$$

where $\boldsymbol{W}(z)$ is the transfer function of the equalizer defined by

$$\boldsymbol{W}(z) = \sum_{k=0}^{M-1} \boldsymbol{W}^{(k)} z^k.$$

The composite system of the two systems is illustrated in the schematic diagram in Fig. 1. All variables can be complex-valued (this is required for such an application using quadrature amplitude modulation (QAM) signals [9]). Substituting (2) into (4), we have

$$\boldsymbol{y}(t) = \boldsymbol{G}(z)\boldsymbol{s}(t) + \boldsymbol{W}(z)\boldsymbol{n}(t), \tag{5}$$

where

$$\boldsymbol{G}(z) := \boldsymbol{W}(z)\boldsymbol{H}(z) = \sum_{k=0}^{K+M-2} \boldsymbol{G}^{(k)} z^k. \tag{6}$$

The blind deconvolution problem can be formulated as follows: Find an equalizer $\boldsymbol{W}(z)$ denoted as $\boldsymbol{W}_1(z)$,

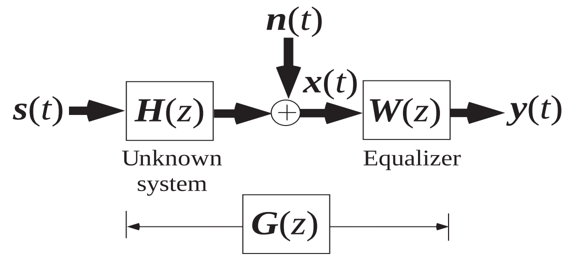

Figure 1: The composite system of an unknown system and an equalizer.

satisfying the following condition, without the knowledge of $\boldsymbol{H}(z)$,

$$\boldsymbol{G}_1(z) := \boldsymbol{W}_1(z)\boldsymbol{H}(z) = \boldsymbol{D}\boldsymbol{P}\boldsymbol{\Lambda}(z), \tag{7}$$

where the composite system is denoted as $\boldsymbol{G}_1(z)$, and $\boldsymbol{D}$ is an $n \times n$ diagonal matrix with diagonal entries d_i, $i = 1, \cdots, n$, $\boldsymbol{P}$ is an $n \times n$ permutation matrix, $\boldsymbol{\Lambda}(z)$ is an $n \times n$ regular diagonal matrix with diagonal entries being monic monomials z^{l_i}, $i = 1, \cdots, n$, where l_i is a non-negative integer. If the composite system $\boldsymbol{G}(z)$ satisfies (7) with $\boldsymbol{G}_1(z) = \boldsymbol{G}(z)$, that is, $\boldsymbol{G}(z)$ has a single nonzero monomial entry in each row and each column, then the composite system is said to be *transparent* [6, 8].

The noise sequence $\{\boldsymbol{n}(t)\}$ is assumed to be Gaussian and uncorrelated with the source signal $\{\boldsymbol{s}(t)\}$. In addition, we assume that its second-order statistics (SOS) are known or can be estimated blindly. Hence, the noise can be removed [1]. For the purpose of analysis, the noise is assumed to be zero, that is, $\boldsymbol{n}(t) = \boldsymbol{0}$.

To find an equalizer $\boldsymbol{W}_1(z)$, we make the following assumptions:

($A1$) The transfer function $\boldsymbol{H}(z)$ in (2) is *equalizable* [6], that is, $rank\boldsymbol{H}(z) = n$ for all nonzero $z \in C$ (this implies that the number of outputs m is greater than or equal to the number of inputs n), where C denotes the set of complex numbers. Moreover, the transfer function $\boldsymbol{H}(z)$ in (2) can be factorized as

$$\boldsymbol{H}(z) = \boldsymbol{H}_I(z)\boldsymbol{U}\boldsymbol{\Lambda}(z), \tag{8}$$

where $\boldsymbol{H}_I(z)$, $\boldsymbol{U}$, and $\boldsymbol{\Lambda}(z)$, respectively, denote an $m \times n$ irreducible matrix, a unitary matrix, and an $n \times n$ regular diagonal matrix with diagonal entries being monic monomials z^{l_i}, $i = 1, \cdots, n$, where l_i is a non-negative integer [6]. Note that an $m \times n$ polynomial matrix $\boldsymbol{X}(z)$ is said to be *irreducible* if $rank\boldsymbol{X}(z) = \min\{m, n\}$ for all $z \in C$ [6].

Equation (8) in the assumption ($A1$) restricts ourselves to a subset of the set of all equalizable systems, but the subset is enough large, because under the condition that the input sequence $\{\boldsymbol{s}(t)\}$ is white, it is shown in [6] that $\boldsymbol{H}(z)$ can be factorized as (8) if the system can be equalized by the SOS of the output sequence $\{\boldsymbol{x}(t)\}$.

($A2$) The input sequence $\{\boldsymbol{s}(t)\}$ is a zero-mean stationary vector process whose component processes $\{s_i(t)\}$ $(i = 1, \cdots, n)$ are temporally non-white and spatially uncorrelated.

From ($A2$), one can calculate the correlation matrix $\boldsymbol{R_s}(\tau)$ of $\{\boldsymbol{s}(t)\}$ as follows:

$$\begin{aligned}\boldsymbol{R_s}(\tau) &= E[\boldsymbol{s}(t+\tau)\boldsymbol{s}^H(t)] \\ &= \text{diag}\{E[s_1(t+\tau)s_1^*(t)], E[s_2(t+\tau)s_2^*(t)], \\ &\qquad \cdots, E[s_n(t+\tau)s_n^*(t)]\}, \quad (9)\end{aligned}$$

where the operator $E[X]$ denotes the expectation of a random matrix or a random variable X and diag$\{\cdots\}$ denotes a diagonal matrix with diagonal elements $\{\cdots\}$. From the sequence $\{\boldsymbol{R_s}(\tau)\}$, the z-transform is calculated, and it is denoted by $\boldsymbol{S_s}(z)$. Note that $\boldsymbol{S_s}(z)$ is also a diagonal matrix diag$\{S_{s_1}(z), \cdots, S_{s_n}(z)\}$. Substituting $e^{-j2\pi f}$ for z in $\boldsymbol{S_s}(z)$, we obtain the power spectral matrix as a matrix function of a frequency f, and it is denoted by $\boldsymbol{S_s}(f) = \text{diag}\{S_{s_1}(f), \cdots, S_{s_n}(f)\}$. For the power spectral elements $S_{s_i}(f)$ $(i = 1, \cdots, n)$, we put the following assumption:

($A3$) The normalized power spectral elements $S_{s_i}(f)/S_{s_i}(0)$ (i=1, $\cdots$, n) are distinct functions (rational in the variable $e^{-j2\pi f}$).

3. IDENTIFIABILITY AND DECONVOLUTION

Now we introduce the following notion for the blind identifiability of an MIMO-FIR system by the SOS of the outputs. It is slightly different from the definition of the blind identifiability made by Hua and Tugnait [3].

Definition: An MIMO-FIR system with the transfer function $\boldsymbol{H}(z) = \boldsymbol{H}_I(z)\boldsymbol{U}\boldsymbol{\Lambda}(z)$ and the power spectrum $\boldsymbol{S_s}(z)$ is said to be *blindly identifiable* up to a permutation matrix, a scaling matrix and a delay matrix by the SOS of the outputs, if any solution pair $\hat{\boldsymbol{H}}(z)$ and $\hat{\boldsymbol{S}}_{\boldsymbol{s}}(z)$ to the following equation

$$\hat{\boldsymbol{H}}(z)\hat{\boldsymbol{S}}_{\boldsymbol{s}}(z)\hat{\boldsymbol{H}}_H(z) = \boldsymbol{H}(z)\boldsymbol{S_s}(z)\boldsymbol{H}_H(z), \qquad (10)$$

subject to all the constraints associated with $\boldsymbol{H}(z)$ and $\boldsymbol{S_s}(z)$, satisfies

$$\begin{aligned}\hat{\boldsymbol{H}}(z) &= \boldsymbol{H}_{I_U}(z)\boldsymbol{P}\boldsymbol{D}\hat{\boldsymbol{\Lambda}}(z) & (11)\\ \hat{\boldsymbol{S}}_{\boldsymbol{s}}(z) &= (\boldsymbol{P}\boldsymbol{D})^{-1}\boldsymbol{S_s}(z)(\boldsymbol{P}\boldsymbol{D})^{-H}. & (12)\end{aligned}$$

where $\boldsymbol{H}_{I_U}(z) = \boldsymbol{H}_I(z)\boldsymbol{U}$, $\boldsymbol{P}$ and $\boldsymbol{D}$ are, respectively, a permutation matrix and a regular diagonal matrix, and $\hat{\boldsymbol{\Lambda}}(z)$ is a regular diagonal matrix with diagonal elements being monic monomials of the variable z.

The following theorem is an extension of Theorem by Hua and Tugnait [3] (for the case of irreducible systems) to the case of equalizable systems satisfying ($A1$).

Theorem 1: An MIMO-FIR system with $m \geq n \geq 2$ is blindly identifiable up to a permutation matrix, a scaling matrix and a delay matrix by the SOS of the outputs, if it satisfies the assumptions ($A1$) through ($A3$).

Proof: Let $\boldsymbol{H}(z)$ and $\boldsymbol{S_s}(z)$ be the transfer function and the power spectrum of the MIMO-FIR system, respectively. From the assumption ($A1$), $\boldsymbol{H}(z)$ can be factored as $\boldsymbol{H}(z) = \boldsymbol{H}_{I_U}(z)\boldsymbol{\Lambda}(z)$, where $\boldsymbol{H}_{I_U}(z) = \boldsymbol{H}_I(z)\boldsymbol{U}$. Now consider any solution pair $\hat{\boldsymbol{H}}(z)$ and $\hat{\boldsymbol{S}}_{\boldsymbol{s}}(z)$ to the following equation:

$$\hat{\boldsymbol{H}}(z)\hat{\boldsymbol{S}}_{\boldsymbol{s}}(z)\hat{\boldsymbol{H}}_H(z) = \boldsymbol{H}(z)\boldsymbol{S_s}(z)\boldsymbol{H}_H(z). \qquad (13)$$

Since $\hat{\boldsymbol{H}}(z)$ satisfies also the assumption ($A1$), it can be factored as $\hat{\boldsymbol{H}}(z) = \hat{\boldsymbol{H}}_{I_U}(z)\hat{\boldsymbol{\Lambda}}(z)$, where $\hat{\boldsymbol{H}}_{I_U}(z) = \hat{\boldsymbol{H}}_I(z)\hat{\boldsymbol{U}}$, $\hat{\boldsymbol{H}}_I(z)$ and $\hat{\boldsymbol{U}}$ are, respectively, an $m \times n$ irreducible matrix and a unitary matrix, and $\hat{\boldsymbol{H}}_{I_U}(z)$ is also irreducible.

We note here that we can easily obtain an extension of the Hua-Tugnait Theorem to the case where the transfer function of an MIMO-FIR system is irreducible and has complex-valued coefficients. Apply this extension to (13), we have

$$\begin{aligned}\hat{\boldsymbol{H}}_{I_U}(z) &= \boldsymbol{H}_{I_U}(z)\boldsymbol{P}\boldsymbol{D}, & (14)\\ \hat{\boldsymbol{S}}_{\boldsymbol{s}}(z) &= (\boldsymbol{P}\boldsymbol{D})^{-1}\boldsymbol{S_s}(z)(\boldsymbol{P}\boldsymbol{D})^{-H}. & (15)\end{aligned}$$

Since $\hat{\boldsymbol{H}}(z) = \hat{\boldsymbol{H}}_{I_U}(z)\hat{\boldsymbol{\Lambda}}(z)$, (14) can be expressed as

$$\hat{\boldsymbol{H}}(z) = \boldsymbol{H}_{I_U}(z)\boldsymbol{P}\boldsymbol{D}\hat{\boldsymbol{\Lambda}}(z). \qquad (16)$$

The equations (16) and (15) respectively correspond to (11) and (12) in the above definition. This completes the proof of Theorem 1.

There comes immediately from Theorem 1 an important result on the blind deconvolution of MIMO-FIR systems with colored inputs using the SOS of the outputs.

Corollary: Let $\{\boldsymbol{y}(t)\}$ be the output sequence of the composite system with $\boldsymbol{n}(t) = \boldsymbol{0}$ in (5) and (6). Suppose that its component sequences $\{y_i(t)\}$ $(i = 1, \cdots, n)$ are mutually uncorrelated. Then the composite system becomes transparent if and only if the power spectral elements $S_{y_i}(z)$ $(i = 1, \cdots, n)$ are equal to the power spectral elements $S_{s_i}(z)$ $(i = 1, \cdots, n)$ up to a permutation and a scaling.

Proof: The necessity is obvious. The sufficiency is shown as follows. From the above assumptions in Corollary, there exist a permutation $\boldsymbol{P}$ and a regular diagonal matrix $\boldsymbol{D}$ such that

$$\boldsymbol{S_y}(z) = \boldsymbol{P}\boldsymbol{D}\boldsymbol{S_s}(z)(\boldsymbol{P}\boldsymbol{D})^H. \qquad (17)$$

Define $\boldsymbol{G}_0(z) := \boldsymbol{PD}$. Then we have

$$\boldsymbol{S_y}(z) = \boldsymbol{G}_0(z)\boldsymbol{S_s}(z)[\boldsymbol{G}_0]_H(z). \quad (18)$$

On the other hand, it follows from (5) with $\boldsymbol{n}(t) = \boldsymbol{0}$

$$\boldsymbol{S_y}(z) = \boldsymbol{G}(z)\boldsymbol{S_s}(z)\boldsymbol{G}_H(z). \quad (19)$$

Therefore, we have the following equation:

$$\boldsymbol{G}(z)\boldsymbol{S_s}(z)\boldsymbol{G}_H(z) = \boldsymbol{G}_0(z)\boldsymbol{S_s}(z)[\boldsymbol{G}_0]_H(z). \quad (20)$$

Since $\boldsymbol{G}_0(z)$ is irreducible, then using Theorem 1, the equation (20) means that there exist a permutation matrix $\hat{\boldsymbol{P}}$, a regular diagonal matrix $\hat{\boldsymbol{D}}$, and a delay matrix $\hat{\boldsymbol{\Lambda}}(z)$ such that

$$\begin{aligned} \boldsymbol{G}(z) &= \boldsymbol{G}_0(z)\hat{\boldsymbol{P}}\hat{\boldsymbol{D}}\hat{\boldsymbol{\Lambda}}(z) & (21)\\ \boldsymbol{S_s}(z) &= (\hat{\boldsymbol{P}}\hat{\boldsymbol{D}})^{-1}\boldsymbol{S_s}(z)(\hat{\boldsymbol{P}}\hat{\boldsymbol{D}})^{-H}. & (22)\end{aligned}$$

Since $\boldsymbol{G}_0(z) = \boldsymbol{PD}$, equation (21) gives

$$\boldsymbol{G}(z) = \boldsymbol{PD}\hat{\boldsymbol{P}}\hat{\boldsymbol{D}}\hat{\boldsymbol{\Lambda}}(z) = (\boldsymbol{P}\hat{\boldsymbol{P}})(\tilde{\boldsymbol{D}}\hat{\boldsymbol{D}})\hat{\boldsymbol{\Lambda}}(z), \quad (23)$$

where $\tilde{\boldsymbol{D}} := \hat{\boldsymbol{P}}^T\boldsymbol{D}\hat{\boldsymbol{P}}$ and it is a diagonal matrix. Because $\boldsymbol{P}\hat{\boldsymbol{P}}$ is a permutation matrix and $\tilde{\boldsymbol{D}}\hat{\boldsymbol{D}}$ is a regular diagonal matrix, the above equation (23) implies $\boldsymbol{G}(z)$ satisfies (7) with $\boldsymbol{G}_1(z) = \boldsymbol{G}(z)$. Therefore the composite system is transparent. This completes the proof of Corollary.

In order to utilize the above corollary for the blind deconvolution of MIMO-FIR systems, we should acquire the knowledge of the power spectral elements $S_{s_i}(z)$, $(i = 1, \cdots, n)$ of the source signal $\{\boldsymbol{s}(t)\}$. However, this restricts the utility of the corollary. On the contrary, the following theorem which is a main contribution of the present paper can be utilized to attain the blind deconvolution of MIMO-FIR systems without the knowledge of the power spectral elements $S_{s_i}(z)$, $(i = 1, \cdots, n)$ of the source signal $\{\boldsymbol{s}(t)\}$.

Theorem 2: Let $\{\boldsymbol{y}(\mathrm{t})\}$ be the output sequence of the composite system with $\boldsymbol{n}(t) = \boldsymbol{0}$ in (5) and (6). Suppose that the transfer function $\boldsymbol{W}(z)$ of the equalizer in (6) is irreducible. Then under $(A1)$ through $(A3)$ the composite system becomes transparent (except for pathological cases) if and only if the power spectral matrix $\boldsymbol{S_y}(z)$ of $\{\boldsymbol{y}(t)\}$ is diagonal.

Remark: When $n < m$, it is almost sure that the transfer function $\boldsymbol{W}(z)$ is irreducible. However, when $n = m$, it is almost sure that $\boldsymbol{W}(z)$ is not irreducible. For a square polynomial matrix $\boldsymbol{W}(z)$, it is irreducible if and only if it is unimodular.

Before presenting a proof of Theorem 2, we provide a lemma, which is an extension of the Hua-Tugnait lemma [3] to the case of transfer functions of complex-valued coefficients.

Lemma 1: Let $\boldsymbol{G}(z)$ be a unimodular matrix with complex-valued coefficients, $\boldsymbol{S_s}(z)$ and $\boldsymbol{S_y}(z)$ each be a regular diagonal power spectral matrix of the same size of $\boldsymbol{G}(z)$, and $\boldsymbol{S_s}(z)$ has distinct diagonal functions up to a constant factor. Then

$$\boldsymbol{G}(z)\boldsymbol{S_s}(z)\boldsymbol{G}_H(z) = \boldsymbol{S_y}(z), \quad (24)$$

if and only if $\boldsymbol{G}(z) = \boldsymbol{PD}$, where $\boldsymbol{P}$ is a permutation matrix and $\boldsymbol{D}$ is a regular diagonal matrix.
Proof: See [3] with appropriate modifications.

Proof of Theorem 2: The necessity is clear. The sufficiency is shown below. It follows from (5) and (6) with $\boldsymbol{n}(t) = \boldsymbol{0}$ that

$$\begin{aligned} \boldsymbol{S_y}(z) &= \boldsymbol{G}(z)\boldsymbol{S_s}(z)\boldsymbol{G}_H(z), & (25)\\ \boldsymbol{G}(z) &= \boldsymbol{W}(z)\boldsymbol{H}(z), & (26)\end{aligned}$$

where $\boldsymbol{H}(z)$ can be factored as

$$\boldsymbol{H}(z) = \boldsymbol{H}_{I_U}(z)\boldsymbol{\Lambda}(z) \quad (27)$$

Let $\boldsymbol{G}_I(z)$ be defined as

$$\boldsymbol{G}_I(z) = \boldsymbol{W}(z)\boldsymbol{H}_{I_U}(z). \quad (28)$$

It follows from (28) that $\boldsymbol{G}_I(z)$ is irreducible (except for pathological cases), because $\boldsymbol{W}(z)$ and $\boldsymbol{H}_{I_U}(z)$ both are irreducible. This means that $\boldsymbol{G}_I(z)$ is unimodular, because $\boldsymbol{G}_I(z)$ is a square polynomial matrix of the constant determinant. It follows from (25) through (28) that

$$\boldsymbol{S_y}(z) = \boldsymbol{G}_I(z)\boldsymbol{S_s}(z)[\boldsymbol{G}_I]_H(z), \quad (29)$$

because

$$\boldsymbol{\Lambda}(z)\boldsymbol{S_s}(z)\boldsymbol{\Lambda}_H(z) = \boldsymbol{S_s}(z). \quad (30)$$

Applying Lemma 1 to the equation (29), we conclude that

$$\boldsymbol{G}_I(z) = \boldsymbol{PD}. \quad (31)$$

Therefore, from (26)-(28) and (31), we have

$$\boldsymbol{G}(z) = \boldsymbol{PD\Lambda}(z), \quad (32)$$

which means that the composite system is transparent. This completes the proof of Theorem 2.

4. BLIND DECONVOLUTION ALGORITHM

Based on Theorem 2, it is seen that the blind deconvolution problem can be solved, if the diagonalization of

$\boldsymbol{S_y}(z)$ is achieved. Therefore, we consider the following function:

$$Q(\boldsymbol{W}) := \int_0^1 F(\boldsymbol{W}, f)df, \tag{33}$$

where $\boldsymbol{W}$ denotes simply the equalizer $\boldsymbol{W}(z)$ in (4), and $F(\boldsymbol{W}, f)$ is defined as

$$F(\boldsymbol{W}, f) = \frac{1}{2}\left\{\sum_{i=1}^{n} log S_{y_{ii}}(f) - log\, det \boldsymbol{S_y}(f)\right\}. \tag{34}$$

Here, let $\boldsymbol{S_y}(z)$ be the z-transform of $\boldsymbol{R_y}(\tau) := \mathrm{E}[\boldsymbol{y}(t+\tau)\boldsymbol{y}^H(t)]$ which is the correlation matrix of $\{\boldsymbol{y}(t)\}$. Then $\boldsymbol{S_y}(f) = [S_{y_{ij}}(f)]$ is obtained by substituting $e^{-j2\pi f}$ for z in $\boldsymbol{S_y}(z)$, where $S_{y_{ij}}(f)$ denotes an element of the matrix $\boldsymbol{S_y}(f)$ and hence $S_{y_{ii}}(f)$ in (34) denotes a diagonal element of the matrix $\boldsymbol{S_y}(f)$. The function $F(\boldsymbol{W}, f)$ is based on the *Hadamard Inequality* [2] and is a non-negative scalar function which takes a minimum (zero) if and only if $\boldsymbol{S_y}(f)$ is diagonal, because $\boldsymbol{S_y}(f)$ is non-negative definite. The non-negative definiteness of the matrix $\boldsymbol{S_y}(f)$ is a well-known fact in the theory of stationary random processes, which is shown for example in [11] (Theorem 7.7 on page 141). The non-negative definiteness of $\boldsymbol{S_y}(f)$ means that the function $F(\boldsymbol{W}, f)$ is a non-negative function [2]. As for any non-negative function in (33), we have the following lemma.

Lemma 2: Let $F(\boldsymbol{W}, f)$ be a non-negative function in (33). Then $F(\boldsymbol{W}, f)$ is zero (almost everywhere) over the interval $[0, 1]$ if and only if $Q(\boldsymbol{W}) = 0$.

In practice, we use the following discrete version of (33):

$$Q(\boldsymbol{W}) := \frac{1}{M}\sum_{k=0}^{M-1} F(\boldsymbol{W}, f_k), \tag{35}$$

where $f_k = \frac{k-1}{M}$, $(k = 1, \cdots, M)$. In the present paper, we treat the discrete version (35) as a criterion function for achieving the blind deconvolution. From Theorem 2 and Lemma 2, one can see that the blind deconvolution can be achieved by minimizing (35). To minimize (35), we use the following rule for calculating the gradient $\nabla_{\boldsymbol{W}^{(l)}} Q$ of the function $Q(\boldsymbol{W})$ with respect to a complex-valued matrix $\boldsymbol{W}^{(l)}$:

$$\nabla_{\boldsymbol{W}^{(l)}} Q = 2\frac{\partial Q(\boldsymbol{W})}{\partial \boldsymbol{W}^{(l)*}}, \tag{36}$$

where $\partial Q(\boldsymbol{W})/\partial \boldsymbol{W}^{(l)*}$ is the partial derivative of $Q(\boldsymbol{W})$ with respect to a complex conjugate matrix $\boldsymbol{W}^{(l)*}$. In deriving the formula (36), we used the following fact: For a scalar function $F(z)$ of a complex variable $z = a+jb$, the partial derivatives with respect to z and z^* are, respectively, defined by

$$\frac{\partial F}{\partial z} = \frac{1}{2}\left(\frac{\partial F}{\partial a} - j\frac{\partial F}{\partial b}\right) \text{ and } \frac{\partial F}{\partial z^*} = \frac{1}{2}\left(\frac{\partial F}{\partial a} + j\frac{\partial F}{\partial b}\right).$$

The right-hand side of the last equation gives the gradient $\nabla_z F$ with respect to a complex variable z. Therefore, $\boldsymbol{W}^{(l)}$ is modified in proportion to (36), that is, the increment $\Delta \boldsymbol{W}^{(l)}$ is given as

$$\Delta \boldsymbol{W}^{(l)} \propto -\nabla_{\boldsymbol{W}^{(l)}} Q \ \left(= -2\frac{\partial Q(\boldsymbol{W})}{\partial \boldsymbol{W}^{(l)*}}\right). \tag{37}$$

Calculating the right hand side of (37) (see [13] for the details), we finally obtain

$$\begin{aligned}\Delta \boldsymbol{W}^{(l)} \ \propto\ & \frac{1}{M}\sum_{k=0}^{M-1} e^{j2\pi f_k l}\{\boldsymbol{S_y}(f_k)^{-1} \\ & -(diag\boldsymbol{S_y}(f_k))^{-1}\}\boldsymbol{W}(f_k)\boldsymbol{S_x}(f_k),\end{aligned} \tag{38}$$

where $diag\boldsymbol{X}$ denotes a diagonal matrix whose elements are the diagonal ones of the matrix $\boldsymbol{X}$, $\boldsymbol{W}(f_k)$ denotes $(\sum_{m=0}^{M-1} \boldsymbol{W}^{(m)} e^{-j2\pi f_k m})$, and $\boldsymbol{S_x}(f_k)$ denotes the power spectral matrix of $\{\boldsymbol{x}(t)\}$. From (38), the up-date rule of $\boldsymbol{W}^{(l)}$ at time t' becomes

$$\begin{aligned}\boldsymbol{W}^{(l)}(t'+1) \ =\ & \boldsymbol{W}^{(l)}(t') + \alpha\Delta\boldsymbol{W}^{(l)} \\ =\ & \boldsymbol{W}^{(l)}(t') + \frac{\alpha}{M}\sum_{k=0}^{M-1} e^{j2\pi f_k l}\{\boldsymbol{S_y}(f_k)^{-1} \\ & -(diag\boldsymbol{S_y}(f_k))^{-1}\}\boldsymbol{W}(f_k)\boldsymbol{S_x}(f_k),\end{aligned} \tag{39}$$

where integer t' denotes an iteration time and α is a small positive constant. The up-date rule (39) is our proposed algorithm for the parameters $\boldsymbol{W}^{(l)}$'s to achieve the blind deconvolution.

5. DISCUSSIONS

In the conventional frequency domain blind deconvolution algorithms (e.g., [5, 10]), $\boldsymbol{W}'(f_k)$ (an $n \times n$ demixing matrix) at each frequency f_k is modified such that $\boldsymbol{W}'(f_k)\boldsymbol{H}'(f_k)$ ($\boldsymbol{H}'(f_k)$ is an $n \times n$ mixing matrix) becomes a matrix $\boldsymbol{PD}$. However, under the condition that the source signals are stationary, the diagonalization of the output spectrum $\boldsymbol{S_y}(f_k)$ at each frequency cannot be applied to finding such a matrix $\boldsymbol{PD}$. On the other hand, in the present paper, since the composite system $\boldsymbol{G}(z)$ can be considered as a unimodular matrix and $\boldsymbol{S_s}(z)$ is assumed to has distinct diagonal functions, the blind deconvolution problem can be solved by the diagonalization of $\boldsymbol{S_y}(z)$.

The proposed algorithm possesses the following attractive feature: Even if the blind deconvolution is implemented in the frequency domain, the permutation problem such as mentioned in [5] does not occur. If $\boldsymbol{W}(f_k)$ is modified by

$$\begin{aligned}\Delta \boldsymbol{W}(f_k) \ \propto\ & \left\{\boldsymbol{S_y}(f_k)^{-1} - (diag\boldsymbol{S_y}(f_k))^{-1}\right\} \\ & \cdot \boldsymbol{W}(f_k)\boldsymbol{S_x}(f_k),\end{aligned} \tag{40}$$

then the permutation problem maybe occurs. [Note that if (34) is differentiated with respect to $\boldsymbol{W}^*(f_k)$, the rule (40) is obtained.] Differently from (40), it can be seen that the right-hand side of (38) is the inverse discrete Fourier transform of the right-hand side of (40). Namely, the proposed algorithm is used to modify coefficient $\boldsymbol{W}^{(l)}$, but it is not used to modify $\boldsymbol{W}(f_k)$ at each frequency f_k. Therefore, since the algorithm (38) possesses the above property and is used to modify $\boldsymbol{W}^{(l)}$, the permutation problem does not occur, even if the algorithm is implemented in the frequency domain.

6. CONCLUSIONS

We have dealt with blind deconvolution problem of MIMO-FIR systems driven by colored source signals. Hua and Tugnait [3] also considered the same problem. Our contributions for the blind deconvolution problem, differently from their contributions, are as follows:

- Blind identifiability of an MIMO-FIR system was shown under a weaker condition (*equalizable* by means of the SOS of the outputs) than the Hua-Tugnait condition (*irreducible*).
- It was shown that the composite system becomes transparent if $\boldsymbol{S}_{\boldsymbol{y}}(z)$ becomes a diagonal matrix under the assumptions $(A1)$ through $(A3)$ and the assumption that $\boldsymbol{W}(z)$ is irreducible. (see Theorem 2)
- Based on Theorem 2, an algorithm in the frequency domain was proposed for solving the blind deconvolution problem.

An attractive feature of the proposed algorithm is that even if the algorithm is implemented in the frequency domain, the permutation problem does not occur.

It has been confirmed by many computer simulations that the proposed algorithm can be used successfully to achieve the blind deconvolution. The simulation results are omitted for the page limit, but can be found in the paper [13].

We have not yet investigated the effects of measurement noise to the performance of the proposed algorithm. This will be treated in our future study.

7. REFERENCES

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