ADAPTIVE BLIND SOURCE SEPARATION IN ORDER SPECIFIED BY STOCHASTIC PROPERTIES

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ABSTRACT

In adaptive blind source separation, the order of recovered source signals is unpredictable due to order indeterminacy. However, it is sometimes desired that the reconstructed source signals be arranged in specified order according to their stochastic properties. To this end, some unsymmetry is introduced to the equilibrium point, and a new unsymmetrical natural gradient algorithm is proposed, which can separate the source signals in desired order. The validity of the proposed algorithm is confirmed through extensive computer simulations.

1. INTRODUCTION

Blind source separation (BSS) has recently attracted much attention, due to its potential applications in various fields, such as wireless telecommunication systems, sonar and radar systems, audio and acoustics, image enhancement and biomedical signal processing (EEG/MEG signals). Generally speaking, the BSS problem consists of recovering mutually independent but otherwise unobserved source signals from their linear mixtures without assuming any a priori knowledge of the original source signals besides certain statistic features [3].

Blind source separation may be implemented off-line or on-line. Efficient batch algorithms are the JADE algorithm of Cardoso et al.[4], the fast fixed-point algorithm of Hyvarinen et al.[8] and the multistage decomposition algorithm of Feng et al.[7]. In this paper, we pay our attention to adaptive algorithms. Typical ones include the natural gradient algorithms of Amari et al.[1], [2], [13], the EASI algorithm of Cardoso et al.[5], the grading learning algorithm of Zhang et al.[14], the nonlinear PCA algorithms of Karhunen et al.[9], the RLS algorithms of Pajunen et al.[11], [15], the cascade neural-network blind signal extraction algorithm of Thawonmas et al.[12], the sequential blind extraction algorithm of Li et al.[10], etc.

In BSS, the order of recovered source signals is unpredictable due to order indeterminacy. However, it is sometimes desired that the reconstructed source signals be arranged in specified order according to their stochastic properties. To the best of our knowledge, none of the aforementioned algorithms can achieve such a purpose. Recently, Cichocki et al.[6] proposed a sequential blind extraction algorithm, which is able to extract signals in decreasing order according to absolute values of their normalized kurtoses, but it can not perform joint recovery of all the source signals.

In this paper, via an order matrix that expresses some unsymmetry, we relate equilibrium points of the BSS algorithm to the order of recovered source signals, and present a new unsymmetrical natural gradient algorithm, which, if the order matrix is properly designed, can separate all the source signals in any order specified by their stochastic properties. Its validity is confirmed by computer simulations.

2. PROBLEM STATEMENT

Consider the noiseless BSS model

$$\mathbf{x}_t = \mathbf{A}\mathbf{s}_t \tag{1}$$

where $\mathbf{A} \in R^{m\times n}$ is called the "mixing matrix", $\mathbf{x}_t = [x_1(t), \cdots, x_m(t)]^T$ is the available vector of observations, and $\mathbf{s}_t = [s_1(t), \cdots, s_n(t)]^T$ is the source vector.

To solve the BSS problem, the following assumptions are made:

A1) the source signals $s_1(t), \cdots, s_n(t)$ are statistically mutually independent;

A2) the unknown mixing matrix $\mathbf{A}$ is fixed and of full column rank;

A3) at most one source signal is normally distributed;

A4) the source signals are zero mean and unit power.

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The goal of BSS is to reconstruct the original source signals using the observations together with the assumptions A1-A4. For this purpose, one usually adjusts an $n \times m$ matrix $\mathbf{W}$, called the "separating matrix", such that the output vector

$$\mathbf{y}_t = \mathbf{W}\mathbf{x}_t \tag{2}$$

is a scaled and/or permuted version of the source vector. That is to say, the combined mixing-separating matrix

$$\mathbf{WA} = \mathbf{\Lambda P} \tag{3}$$

is the product of an invertible diagonal matrix $\mathbf{\Lambda}$ and a permutation matrix $\mathbf{P}$, namely, a generalized permutation matrix.

3. BSS IN SPECIFIED ORDER

To find the optimal separating matrix $\mathbf{W}$ in (3), a variety of algorithms have been proposed. Among these algorithms, the natural/relative gradient algorithms work very efficiently, and thus widely used in practice. The basic form is given by [2]

$$\mathbf{W}_{t+1} = \mathbf{W}_t + \eta_t \left[\mathbf{I} - \mathbf{\Phi}(\mathbf{y}_t)\mathbf{y}_t^{\mathsf{T}}\right] \mathbf{W}_t \tag{4}$$

where η_t is a learning rate, and $\mathbf{\Phi}(\mathbf{y}_t) = [\phi_1(y_{1,t}), \cdots, \phi_n(y_{n,t})]^{\mathsf{T}}$ denotes a vector of nonlinearly-modified output signals.

An efficient learning algorithm for BSS should have two key properties. One is the property of keeping the separating matrix from becoming singular, which is the premise that the BSS algorithm works. The other is the equivariant property [5], which offers uniform performance of the BSS algorithm, meaning whose behaviors are independent of the mixing matrix $\mathbf{A}$.

It has been showed [5], [13] that the algorithm (4) is equivariant and satisfies the invertibility requirement of the separating matrix. Therefore, we focus on it in this paper to develop the BSS algorithm with desired order of source signals.

From the model (1), it can be seen clearly that there are two indeterminacies in BSS: scaling ambiguity and permutation ambiguity. Exploiting these two indeterminacies, one can reconstruct the source signals in any amplitude and in any order. As an example, the recovered source signals obtained by the algorithm (4) have such scales that

$$E\{\phi_i(y_i)y_i\} = 1, \quad i = 1, \cdots n \tag{5}$$

while the order of the reconstructed source signals is unpredictable.

In some applications of BSS, it is desired that certain source signal of interest appears at certain position, that is, the recovered source signals should be arranged in order specified by some properties. To this end, Cichocki et al. [6] proposed a sequential blind extraction algorithm, which is able to extract signals in decreasing order according to absolute values of their normalized kurtoses. When the number of source signals is large and only some of them are of interest, the algorithm is useful. However, such an algorithm can not perform joint recovery of all the source signals. It can not make the source signals extracted in increasing or other desired order according to the kurtosis. If the source signal of interest has the smallest absolute kurtosis, then it will take much time to recover it. Additionally, it can not achieve extraction of source signals in order specified by other stochastic properties such as the fourth-order moment.

Obviously, it is impossible to separate the source signals in desired order by applying the algorithm (4), since there are no relationship between the equilibrium points and the order of the recovered source signals. Based on this consideration, we propose the following unsymmetrical natural gradient algorithm:

$$\mathbf{W}_{t+1} = \mathbf{W}_t + \eta_t \left[\mathbf{I} - \mathbf{\Xi} - \mathbf{\Phi}(\mathbf{y}_t)\mathbf{y}_t^{\mathsf{T}}\right] \mathbf{W}_t \tag{6}$$

where $\mathbf{\Xi}$ is a matrix, whose on-diagonal elements are zeros, and off-diagonal elements depend on the difference between the actual order and the desired one of the recovered source signals. For convenience of statement, we call $\mathbf{\Xi}$ the order matrix in this paper.

The equilibrium points of the algorithm (6) satisfy

$$E\{\phi_i(y_i)y_j\} = \delta_{ij} - \Xi_{ij}, \quad i,\ j = 1, \cdots n \tag{7}$$

where δ_{ij} is the Kronecker function, which equals unit when $i = j$, and otherwise zero. Clearly, the equilibrium points are now related to certain properties of the recovered source signals. Therefore, the algorithm (6) can be expected to achieve BSS in desired order if the order matrix $\mathbf{\Xi}$ is properly designed. See next section for an example of the design of $\mathbf{\Xi}$.

About the algorithm (6), we remark that it achieves blind separation in two stages. The first stage is a capture stage, in which the order matrix $\mathbf{\Xi}$ works, and the source signals are coarsely reconstructed and captured in specified order. Once all the source signals are arranged in desired order, the matrix $\mathbf{\Xi}$ becomes a null matrix, and the algorithm goes to the second stage. Since (6) degenerates to the general natural gradient algorithm (4), the output signals are processed as independent as possible, and thus this stage, to some extent, can be regarded a fine separation stage.

4. COMPUTER SIMULATIONS

In order to verify the effectiveness of the unsymmetrical natural gradient algorithm (6) for BSS, we consider the separation of the following four source signals (taken from [1], [13]):

$$\mathbf{s}_t = \begin{bmatrix} \operatorname{sign}(\cos(2\pi 155t)) \\ \sin(2\pi 90t) \\ \sin(2\pi 300t + 6\cos(2\pi 60t)) \\ n(t) \end{bmatrix} \tag{8}$$

where $n(t)$ is a random source uniformly distributed in $[-1, +1]$. The above four sources are all sub-Gaussian signals, with the normalized kurtosis equal to -2, -1.5, -1.05 and -1.2, respectively. In simulations, the mixing matrix $\mathbf{A}$ is fixed in each run, but the elements of which are randomly assigned in the range $[-1, +1]$. The mixed signals are sampled at the rate of 10KHz.

To evaluate the performance of the BSS algorithm, we use the cross-talking error [1], [13]

$$\begin{aligned} \mathrm{E}_{ct} &= \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{|c_{ij}|}{\max_k |c_{ik}|}-1\right) \\ &\quad + \sum_{j=1}^{n}\left(\sum_{i=1}^{n}\frac{|c_{ij}|}{\max_k |c_{kj}|}-1\right) \end{aligned} \tag{9}$$

where $\mathbf{C} = \mathbf{WA} = \{c_{ij}\}$ is the combined mixing - separating matrix. To know how many source signals are recovered in specified order, we use the cross-arranging number

$$\mathrm{N}_{cr} = \sum_{i=1}^{n} d_i \tag{10}$$

where $d_i = 0$ if the i-th source signal is recovered at desired position, and otherwise $d_i = 1$.

The order matrix $\mathbf{\Xi}$ plays an important role in the unsymmetrical natural gradient algorithm (6), and it must be designed carefully. Suppose that the recovered source signals should be arranged in increasing order according to absolute value of the normalized kurtosis, it is the simplest way to select the off-diagonal element Ξ_{ij} as difference between the actual absolute value and the desired one of the estimated normalized kurtosis of the i-th recovered source signal. It should be pointed out that if the sources consist of mixed sub-Gaussian and super-Gaussian signals, then different nonlinearities should be used to perform the BSS, and the design of $\mathbf{\Xi}$ becomes more complicated, which remains unsolved in this paper.

In simulations, the learning rate is taken as $\eta = 50 \times T$, where $T = 10^{-4}$ is the sampling period, and the same nonlinearity, i.e., the cubic function $\phi_i(y_i) = y_i^3$ is used for the four sub-Gaussian source signals. Taking 20000 samples, we simulated 500 independent runs (in each run, both the mixing matrix $\mathbf{A}$ and the noise signal $n(t)$ are regenerated randomly). The average cross-talking error and cross-arranging number are plotted in Fig.1, while Figs.2-4 show respectively, the cross-talking error and cross-arranging number, the original source signals and the recovered source signals of a typical run. From Figs.1-4, we can clearly see that the proposed algorithm (6) can achieve BSS in desired order, and the separation process is performed in two stages. In the first stage, all the source signals are coarsely reconstructed and captured in specified order according to their stochastic properties. The second stage is the fine separation stage, in which the signals are processed as independent as possible.

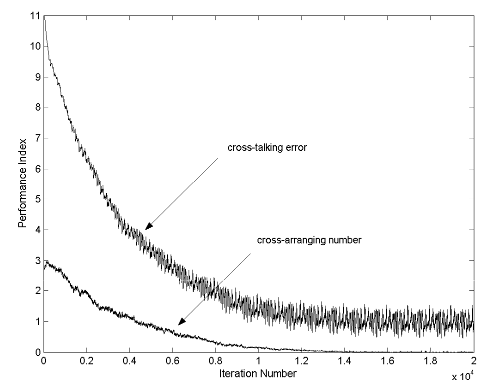

Fig.1 Average cross-talking error and average cross-arranging number over 500 independent runs.

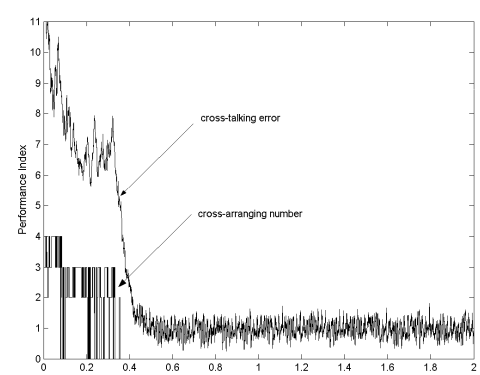

Fig.2 The cross-talking error and the cross-arranging number of a typical run.

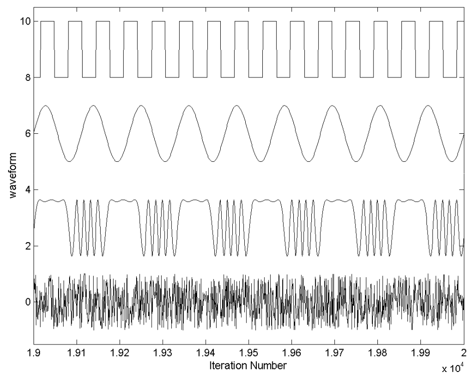

Fig.3 Waveforms of the original source signals of a typical run.

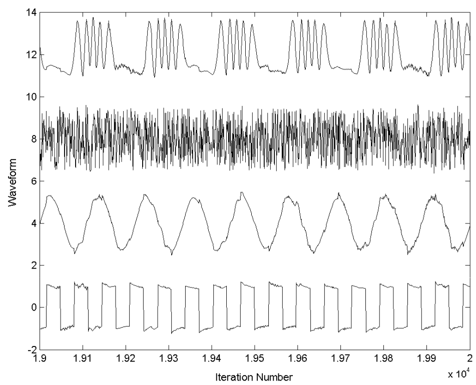

Fig.4 Waveforms of the recovered source signals of a typical run.

5. CONCLUSIONS

Via the order matrix that expresses some unsymmetry, we relate the equilibrium points to the order of recovered source signals, and present a new unsymmetrical natural gradient algorithm, which can not only perform the BSS, but also arrange the recovered source signals in desired order. The validity of the proposed algorithm is confirmed by computer simulations.

6. REFERENCES

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