# BLIND EQUALIZATION OF FRACTIONALLY-SPACED CHANNELS

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## ABSTRACT

We approach the problem of blind identification and equalization (BIE) of single-user digital communication channels from the perspective of blind source separation (BSS). A new BSS-based BIE algorithm is proposed in this paper and is compared with a subspace method as well as a normalized variant of the well-known constant modulus algorithm (NCMA). The equalization qualities of the three algorithms are assessed using channels with well-conditioned and ill-conditioned convolution matrices. It is found that the BSS-based algorithm outperforms the other algorithms except for short source data sequences. The subspace method, which inverts the estimated channel to obtain the equalizer, leads to poor results in the case of the ill-conditioned channel. The simple NCMA suffers from slow convergence or misconvergence except for well-conditioned channels of low order.

## 1. INTRODUCTION

In digital communications, a number of propagation effects (such as multipath propagation in wireless environments or dispersion in optical fibre) cause distortion of the transmitted signal at the receiving end. Equalization techniques must be used to recover the original data from the distorted received signal. Traditionally, training sequences known by the receiver were employed to aid in the deconvolution process. However, operating 'blindly' makes a more efficient use of bandwidth resources, as no periodical transmission of a training sequence is required.

Due to the cyclostationary nature of digital signals, when the received signal is fractionally sampled (in time and/or space) the blind channel identification and equalization (BIE) problem accepts a blind source separation (BSS) model of instantaneous linear mixtures [1]. The BSS approach proves specially attractive because source separation methods with the equivariance property [2] guarantee a robust blind equalization even in channels with deep notches and ill-conditioned convolution matrices. However, the inherent source scale and permutation indeterminacy in BSS precludes a successful blind identification of the channel.

The present contribution proposes a simple post-processing stage which allows blind equalization and channel estimation based on BSS techniques. Through a number of computer simulations in a variety of channel conditions, the BSS approach is compared with the subspace method of [3]. Although these two methods process the received signal in sample blocks (batch processing), they are also compared to the widespread normalized constant modulus algorithm (NCMA) of [4], which operates in an adaptive (i.e., time recursive) fashion. All of the blind equalization methods investigated in this work use fractional sampling, which, theoretically and in the absence of noise, allows perfect signal reconstruction using a finite impulse response (FIR) equalizer filter [5].

After developing the signal model in Section 2, the BSS-based approach is presented in Section 3. For the sake of completeness, the subspace method and the NCMA are briefly reviewed in Section 4. Then, Section 5 describes the simulation environment used in Section 6 to compare the performance of the three blind equalization methods. The results are discussed in Section 7, and conclusions are drawn in Section 8.

*Notations:* Symbols $(.)^T$ and $(.)^H$ denote the transpose and Hermitian (conjugate-transpose) matrix operators, $(.)^*$ denotes complex conjugation, E[.] is the mathematical expectation, and $\delta(n)$ is Kronecker's delta function.

## 2. PROBLEM STATEMENT AND SIGNAL MODEL

Let $x(k)$ be a sequence of non-Gaussian data symbols, which are assumed to be zero mean, independent and identically distributed (i.i.d.), with autocorrelation function $R_x(n) = \mathrm{E}[x(k)x^*(k-n)] = \delta(n)$. The data sequence is transmitted at a known baud rate $1/T$ through a channel with impulse response $h(n)$ spanning $(M+1)$ data symbols.

$x(n)$ $y(n)$ $u(n)$ $\hat{x}(n)$ $\tilde{x}(n)$

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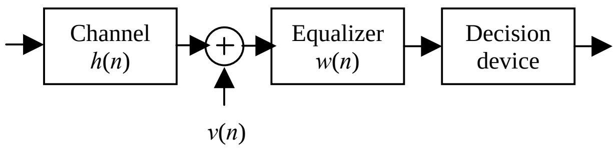


**Fig. 1** Blind equalizer in a digital communication system.

We assume, without loss of generality, that a single sensor is oversampled by an integer factor of $P$. The received baseband signal is then given by $u(n) = y(n) + v(n)$, where $y(n) = \sum_k x(k)h(n - kP)$ is the noise-free channel output and $v(n)$ denotes the zero-mean additive sensor noise.

The objective of blind channel identification is to estimate the channel impulse response $h(n)$ from the only observation of the receiver output $u(n)$. Similarly, blind channel equalization is concerned with the estimation of the data sequence $x(n)$ from $u(n)$ by using an equalizer filter $w(n)$, perhaps obtained from the inversion of a channel filter estimate if channel identification is previously carried out. The BIE system model is graphically depicted in Fig. 1.

Consider an observation window of $N$ samples, $N$ being a multiple of the oversampling rate. Denote $u^{(i)}(n) = u(nP + i)$, $i = 0, 1, ..., P$-1, with analogous notations for $h^{(i)}(n)$ and $v^{(i)}(n)$. Stacking $N/P$ consecutive values of the received signal in vector $\boldsymbol{u}^{(i)}_n = [u^{(i)}(n), ..., u^{(i)}(n - N/P + 1)]^T$, we have that

$$\boldsymbol{u}^{(i)}_n = \boldsymbol{H}_i \boldsymbol{x}_n + \boldsymbol{v}^{(i)}_n ,$$

in which $\boldsymbol{x}_n = [x(n), ..., x(n - M - N/P + 1)]^T$, and $\boldsymbol{H}_i$ is the $N/P$ x $(M + N/P)$ Toeplitz convolution matrix associated with the filter $[h^{(i)}(0), ..., h^{(i)}(M)]^T$. Then:

$$\boldsymbol{u}_n = \boldsymbol{H}\, \boldsymbol{x}_n + \boldsymbol{v}_n \qquad (1)$$

where $\boldsymbol{u}_n = [\boldsymbol{u}^{(0)}{}_n{}^T, \ ... \ , \boldsymbol{u}^{(P-1)}{}_n{}^T]^T$ (and similarly for $\boldsymbol{v}(n)$), and $\boldsymbol{H} = [\boldsymbol{H}_0{}^T, ..., \boldsymbol{H}_{P-1}{}^T]^T$ represents the $N$ x $(M + N/P)$ channel filtering matrix, which is assumed full column rank. Sufficient conditions for $\boldsymbol{H}$ to be full column rank are given by the convolution matrix rank theorem [5].

### 3. BSS-BASED BLIND EQUALIZATION

Eqn. (1) corresponds to the BSS model of instantaneous linear mixtures [6]. The i.i.d. source assumption makes the source components in eqn. (1) statistically independent. Hence, BSS methods based on higher-order statistics (HOS) can be used to recover the source symbol vector $\boldsymbol{x}_n$ [1]. However, the BSS problem presents an inherent indeterminacy related to the scale (phase) and ordering of the sources, which in a general separation scenario are usually unimportant. By contrast, in the BIE model (1) the arrangement and scale of the recovered sources is crucial, for they may alter the time sequence of the data symbols as well as the temporal structure of the channel impulse response. Hence, the solution obtained via BSS needs to be refined if it is to be useful in the BIE problem, specially for channel identification purposes.

The following post-BSS stage overcomes the order and scale indeterminacies in the source vector $\boldsymbol{x}'_n$ estimated via BSS to obtain an adequate estimate of the channel matrix $\hat{\boldsymbol{H}}$ and its corresponding source symbol estimate $\hat{\boldsymbol{x}}_n$.

1. Estimate the cross correlation between a reference element $x'_{ref}$ and all remaining elements $x'_i$: $R_{x'_{ref},x'_i}(\rho) = E\left[x'_{ref}(n)x'^{*}_i(n-\rho)\right]$ where $x'_i(n)$ denotes the $i$th element of $\boldsymbol{x}'_n$ at time $n$.
2. Obtain the lag $\hat{\rho}_i$ with maximum absolute value of the cross correlation $R_{x'_{ref},x'_i}(\rho)$ and obtain the value $\tilde{n}_i$ of the cross correlation at lag $\hat{\rho}_i$.
3. Multiply element $x'_i$ by $\tilde{n}_i$; divide the $i$th column of $\hat{\boldsymbol{H}}$ by $\tilde{n}_i$.
4. Rearrange the elements of $\boldsymbol{x}'_n$ and the columns of $\hat{\boldsymbol{H}}$ according to descending order of $\hat{\rho}_i$.

Utilizing a signal-quality criterion based on the normalized kurtosis, the least noisy component among the estimated sources is chosen as a reference element. Considering real-valued signals, the normalized kurtosis $k_i$ of the estimated source $i$ is defined as

$$k_i = \frac{E\left[x'_i(n)^4\right]}{E^2\left[x'_i(n)^2\right]} - 3 \qquad (2)$$

In digital communications, source symbols usually have non-Gaussian (typically sub-Gaussian) distributions while the additive noise can often be assumed to be Gaussian. In these conditions, lower noise levels will produce higher absolute values of normalized kurtosis.

In the simulations reported later in this paper we use the BSS method of [7] (see also [8]), which is an efficient approximate closed-form solution for the maximization of the sum of squared fourth-order cumulants (MaSSFOC) at the separator output. Note that this choice of BSS method is somewhat arbitrary, since we are primarily concerned with the application of BSS as a general strategy.

## 4. SUBSPACE METHOD AND NCMA

### 4.1. Subspace method

The method presented in [3] exploits the orthogonality between the signal and noise subspaces of the observed vector (1) to estimate the coefficients of the channel impulse response up to a multiplicative constant. The method uses second-order statistics (SOS) only, as opposed to HOS typically employed in BSS methods. In the presence of noise, the subspace method was implemented using a quadratic least-squares cost function. By taking advantage of the channel Toeplitz structure, the optimization of this cost function was smartly reduced to a simple eigenvalue decomposition. After identifying the channel, an FIR linear decorrelating (or zero-forcing) equalizer $\boldsymbol{W}$ was obtained from the pseudo-inverse of the estimated channel matrix:

$$\boldsymbol{W} = \hat{\boldsymbol{H}}^H \boldsymbol{S} diag\left[\left(\lambda_0 - \sigma^2\right)^{-1}, \ldots, \left(\lambda_{M+N/P-1} - \sigma^2\right)^{-1}\right] \boldsymbol{S}^H \tag{3}$$

where $\hat{\boldsymbol{H}}$ is the estimated channel filtering matrix, the columns of $\boldsymbol{S}$ contain the signal-subspace eigenvectors of the sensor autocorrelation matrix, $\lambda_i$ denote their associated eigenvalues and $\sigma^2$ represents the noise variance. Note that in our notation $N$ denotes the window size in samples, i.e., the product of the stacking level times the oversampling factor, as opposed to the stacking level of [3].

### 4.2. NCMA

The constant modulus algorithm (CMA) sets the equalizer coefficients to minimize the envelope variation of the equalized signal. It was originally proposed in [4] and uses a stochastic gradient search to minimize the CM cost function. Due to its simplicity and generally good equalization performance the CMA is widely used [6]. A normalized version (NCMA), presented in [9], is implemented for our comparison experiments. This normalized version obtains faster convergence and guarantees stability compared to the original algorithm in [4]. The filter coefficient update equation is given by:

$$\boldsymbol{w}_{n+1} = \boldsymbol{w}_n - \frac{\boldsymbol{u}_n}{\left\|\boldsymbol{u}_n\right\|^2 + \varepsilon}\left(1 - \frac{1}{\left|\hat{x}(n)\right|}\right)\hat{x}^*(n) \tag{4}$$

where $\boldsymbol{w}_n = [w_0, \ldots, w_{N-1}]^T$ denotes the current and $\boldsymbol{w}_{n+1}$ the updated equalizer filter vectors, $\boldsymbol{u}_n = [u(n), \ldots, u(n-N+1)]^T$ is the observed vector, $\hat{x}(n)$ the equalizer output and $\|.\|$ denotes the L-2 norm. Parameter $\varepsilon$, typically set to a small value, is used to avoid very large step sizes which can result in increased noise at the equalizer output when the input signal $u(n)$ has a low amplitude. The window size $N$ corresponds to the length of the filter vector $\boldsymbol{w}$. The equalizer output is obtained as:

$$\hat{x}(n) = \boldsymbol{w}_n^H \boldsymbol{u}_n \tag{5}$$

Due to the nature of its cost function, the CMA uses HOS implicitly.

## 5. SIMULATION ENVIRONMENT

Performance tests are carried out to compare the behaviour of the three blind equalization methods in a digital communications setting. A binary phase shift keying (BPSK) signal $x(n)$ is oversampled by a factor $P$=2 (unless otherwise stated) and transmitted through the fractionally spaced channels as depicted in Fig. 2. The source signal has an equal probability of the symbol states, is i.i.d. and has unit variance. These signal characteristics fulfill the requirements of the blind equalization methods in [3] and [9], although the i.i.d. assumption is not strictly required in these two methods. Simulations are carried out with $Ne$ = 120, 240, 500, 1000 and 2500 observed symbol periods. Gaussian noise $v(n)$ is added to the channel output $y(n)$ to obtain the received signal $u(n)$. Perfect symbol timing recovery is assumed throughout.

The equalized symbols $\hat{x}(n)$ are first scaled and time shifted before they are compared with the true source symbols $x(n)$. The mean square error (MSE)

$$MSE\,(dB) = 10 \log_{10}\left\{\frac{1}{k}\sum_{n=1}^{k}\left|\hat{x}(n) - x(n)\right|^2\right\} \tag{6}$$

is used as a performance criterion, where $k$ denotes the number of equalized symbols. In the case of the BSS-based and the subspace methods the MSE is calculated from all available equalized symbols. In the case of the NCMA, only the last 100 equalized symbols are used, before which convergence is expected to have taken place. Two different channels are considered. Both have deep notches and are non-minimum phase channels but one channel has an ill-conditioned convolution matrix while the other has a well-conditioned convolution matrix.

$x(n)$ $y(n)$ $u(n)$ $\hat{x}(n)$

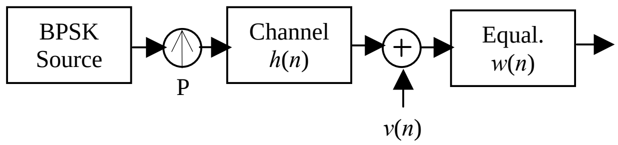


**Fig. 2** Performance test setup.

Below results have been obtained by averaging 100 Monte Carlo runs.

The channels, which are of order 7, model a severe frequency-selective transmission medium. The condition number of the associated convolution matrix $\boldsymbol{H}$ is 365 for the well-conditioned channel and 8886 for the ill-conditioned channel at $N$ = 20. The condition number is defined as the ratio of the largest singular value to the smallest singular value of $\boldsymbol{H}$. The channels have the fractionally spaced impulse responses ($P$ = 2):

$h_1$ = [0.1500 0.3679 0.0611 0.3413 -0.0170 0.3066 -0.0322 0.3372 -0.0414 0.5448 0.2078 0.1967 0.0659 0.0690 0.0379 0.0213]
$h_2$ = [0.1500 0.3679 0.0611 0.0180 -0.0170 0.5438 -0.0322 0.0544 -0.0414 0.4623 0.2078 0.1722 0.0659 0.0535 0.0379 0.0224]

where $h_1$ and $h_2$ are the well-conditioned and the ill-conditioned channel impulse responses, respectively.

The signal-to-noise ratio (SNR) is defined as

$$SNR\ (dB) = 10 \log_{10} \frac{E\left[ y(n)^2 \right]}{E\left[ v(n)^2 \right]} \qquad (7)$$

A common requirement of the three blind equalization methods is that the channel convolution matrix $\boldsymbol{H}$ be full column rank. This condition is fulfilled for the window sizes shown in Table 1.

**Table 1** Window sizes used for simulations.

| Algorithm | Window size, $N$ |
|---|---|
| NCMA | 10 |
| Subspace method | 20 |
| BSS-based method | 20 |

## 6. SIMULATION RESULTS

### 6.1. Well-conditioned channel

Fig. 3 shows the MSE obtained with the well-conditioned channel as a function of the number of observed symbol periods, $Ne$. The subspace method obtains similar results as the BSS-based method at a low number of symbols, while the BSS-based method improves with longer data sequences due to the exploitation of HOS, which require more data for accurate estimation than SOS. The NCMA performance improvement with increasing number of symbols is slower than that of the other algorithms.

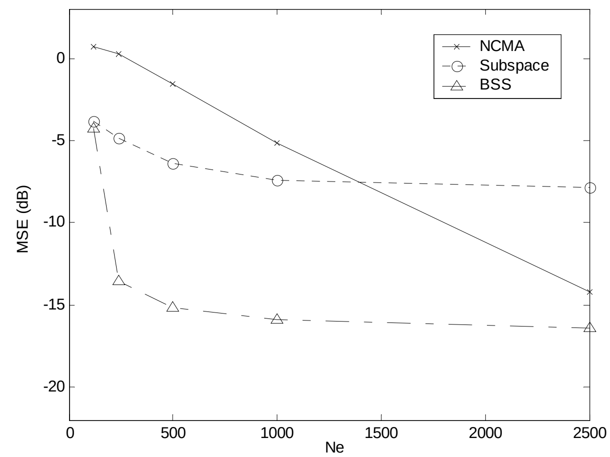


**Fig. 3** MSE vs. $Ne$, SNR = 25 dB.

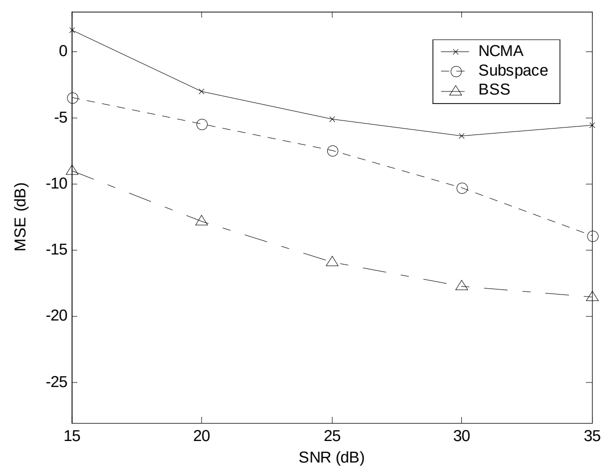


**Fig. 4** MSE vs. SNR, $Ne$ = 1000.

Fig. 4 shows the MSE as a function of the SNR. It can be observed that at low SNR values the BSS-based method still obtains the lowest MSE. The MSE of the NCMA is nearly 3dB at SNR = 15dB. This upper limit is reached when convergence, and thus equalization, fails.

### 6.2. Ill-conditioned channel

Fig. 5 shows that all algorithms perform worse compared to the well-conditioned channel. The worse performance of the subspace and BSS methods can be explained with

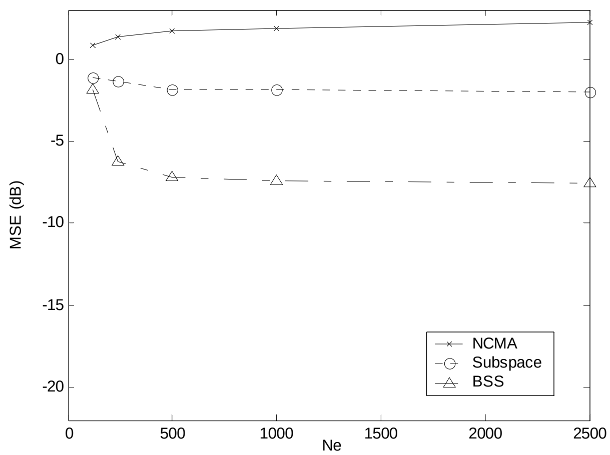


**Fig. 5** MSE vs. *Ne*, SNR = 25 dB.

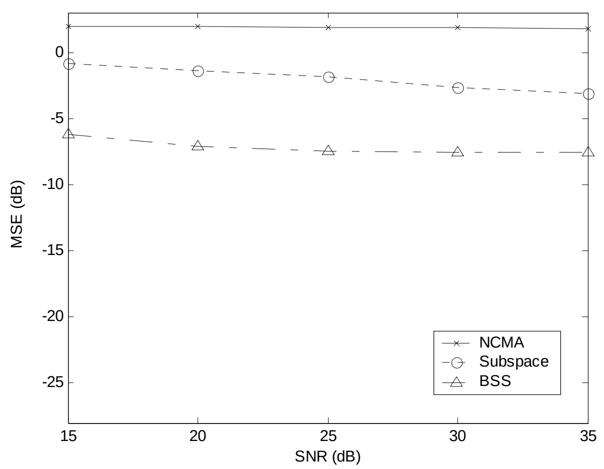


**Fig. 6** MSE vs. SNR, *Ne* = 1000.

the higher condition number of the channel convolution matrix ***H***. The subspace method uses inversion of the channel estimate, which is more sensitive to estimation errors for higher condition numbers of ***H***. Although the BSS method does not use matrix inversion to estimate the source symbols, the higher condition number of ***H*** - which is in fact the mixing matrix in the BSS setting - leads to a worse source separation quality due to the presence of noise. In effect, the uniform performance property of equivariant BSS techniques is only expected to hold strictly in the noiseless case [2]. Still, the BSS-based method obtains a lower MSE than the subspace method. It can be seen that the equalization with the NCMA failed. However, it was observed that the NCMA performed reasonably well when the equalizer filter of the NCMA was initialised close to an optimum setting.

Fig. 6 shows the MSE as a function of the SNR. The MSE of the subspace method and the BSS-based method are nearly independent of the SNR value. Equalization with the NCMA fails even at high SNR values.

### 6.3. Further simulations

Simulations were also carried out with a channel of order 3 without deep notches. The window size of the NCMA was $N$ = 4 in this case while the other window sizes were as defined in Table 1. The results showed that the MSE of the subspace method was about 5 dB lower in all simulations while the MSE of the BSS-based method was roughly the same. This highlights the fact that the BSS-based algorithm is robust with respect to the structure of the channel convolution matrix ***H***. The NCMA obtained a faster convergence compared to the well-conditioned channel and obtained the lowest MSE for long data sequences.

Next, the oversampling factor was doubled. The window size of the NCMA was $N$ = 8, while the window sizes of the subspace method and the BSS-based method were $N$ = 40. It was found that with $P$ = 4 the subspace and the BSS-based methods obtained similar results as with $P$ = 2 at half the number of observed symbol periods. The NCMA obtained about a 12 dB lower MSE for short data sequences.

### 6.4. Computational requirements

The computational cost of the equalization methods was compared by counting the number of floating point operations (flops). The results were obtained under the following conditions: well-conditioned channel, *Ne* = 1000 symbol periods and SNR = 25 dB.

**Table 2** Computational requirements.

| Algorithm | Flops (x$10^6$) |
|---|---|
| NCMA | 0.1 |
| Subspace method | 4.7 |
| BSS-based method | 22.9 |

It can be seen in Table 2 that the BSS method, which performed best in all simulations, had the highest computational requirements. For the algorithms analyzed in this work, it emerges that the computational requirements are linked to their performance: the higher the computational requirements, the better the performance.

## 7. DISCUSSION

The simulation results show the superiority of the BSS-based equalization algorithm in most situations, except for very short data sequences. The performance breakdown at short sample size is due to the larger amounts of data required to obtain accurate HOS estimates. It is observed that the BSS-based method achieves the best equalization for most simulated SNR ratios, which is certainly related to the robustness of HOS to Gaussian noise. The BSS method obtains the same equalization quality for channels with and without deep notches as long as they are well-conditioned. This is in line with the theory: the performance of equivariant BSS methods is independent, at least in the noiseless case, of the structure of the mixing matrix, which corresponds to the channel convolution matrix in the BIE problem. However, the good performance of the BSS method comes at the expense of higher computational requirements.

The subspace method performs worse than the BSS method except at very short data sequences. Since in the subspace method the estimated channel convolution matrix is inverted to obtain the equalizer, it is expected to perform worse with the ill-conditioned channel than the BSS-based method, which directly estimates the source symbols. The use of SOS in the subspace method leads to lower MSE values for short sample length compared to the BSS method. The performance worsens in the case of channels with deep notches, as relatively long FIR filters are necessary to equalize such channels. The use of channel inversion can lead to worse equalization than obtained via BSS and in some cases also worse than that of NCMA. Yet, the method achieves reasonable equalization when the channel is well-conditioned.

The NCMA performs well only with the channel of order 3. In all other cases either the convergence rate is very low (well-conditioned channel) or the algorithm does not converge at all (ill-conditioned channel) due to an unsuitable equalizer filter initialization. However, the advantage of the NCMA is its simple implementation and low computational requirements.

Interestingly, doubling the oversampling factor improves the performance of all investigated methods.

## 8. CONCLUSIONS

This paper has addressed the problem of blind identification and equalization of digital communication channels. We have put forward a simple postprocessing stage which enables channel identification from the application of a BSS method. This technique has been compared, through a variety of simulations, with two other blind equalization algorithms, namely, the subspace method and the NCMA.

In most cases the BSS-based approach has been found to outperform the other methods. The most attractive benefit of the BSS approach is its robustness to ill-conditioned channels, at least in high SNR scenarios. The subspace method proves suitable in rapidly changing environments, such as fast fading wireless channels. The BSS-based method, however, requires longer data sequences to obtain good equalization due to the use of HOS, which also entails an increased computational burden. The NCMA suffers from slow convergence except for short equalizer filters, which can only be used in low-order channels. Thus, the NCMA is suitable in stationary channels where the slow convergence rate does not pose a problem and when convergence can be guaranteed.

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